CHAPTER 4

LEVEL FLIGHT PERFORMANCE

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FIGURES

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EQUATIONS

$L - W + T_G \sin \alpha_j = 0$ (Eq 4.13) 4.9

$L = W - T_G \sin \alpha_j$ (Eq 4.14) 4.9

$C_L = \frac{L}{qS}$ (Eq 4.15) 4.9

$C_L = \frac{W - T_G \sin \alpha_j}{qS}$ (Eq 4.16) 4.9

$D = C_{D_P} qS + \frac{\left(W - T_G \sin \alpha_j\right)^2}{\pi e AR qS}$ (Eq 4.17) 4.9

$D = C_{D_P} qS + \frac{W^2}{\pi e AR qS}$ (Eq 4.18) 4.9

$q = \frac{1}{2} \rho_{ssl} V_e^2$ (Eq 4.19) 4.9

$q = \frac{1}{2} \rho_a V_T^2$ (Eq 4.20) 4.9

$q = \frac{1}{2} \gamma P_a M^2$ (Eq 4.21) 4.10

$D = \frac{C_{D_p} \rho_{ssl} V_e^2 S}{2} + \frac{2 W^2}{\pi e AR S \rho_{ssl} V_e^2}$ (Eq 4.22) 4.10

$D = \frac{C_{D_p} \rho_a V_T^2 S}{2} + \frac{2 W^2}{\pi e AR S \rho_a V_T^2}$ (Eq 4.23) 4.10

$D = \frac{C_{D_p} \gamma P_a M^2 S}{2} + \frac{2W^2}{\pi e AR S \gamma P_a M^2}$ (Eq 4.24) 4.10

$D = \dfrac{C_{D_{p\,(M)}}\,\gamma\,P_a\,M^2\,S}{2} + \dfrac{2W^2}{\pi\,e_{(M)}\,AR\,S\,\gamma\,P_a\,M^2}$	(Eq 4.25)	4.12
$\dfrac{D}{\delta} = \dfrac{C_{D_{P(M)}}\,\gamma\,P_{ssl}\,M^2\,S}{2} + \dfrac{2\,(W/\delta)^2}{\pi\,e_{(M)}\,AR\,S\,\gamma\,P_{ssl}\,M^2}$	(Eq 4.26)	4.12
$\dfrac{D}{\delta} = f\left(M, \dfrac{W}{\delta}\right)$	(Eq 4.27)	4.13
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$\dfrac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(M, \dfrac{N}{\sqrt{\theta}}, R_e\right)$	(Eq 4.29)	4.14
$\dfrac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(M, \dfrac{N}{\sqrt{\theta}}\right)$	(Eq 4.30)	4.15
$\dfrac{T_{N_x}}{\delta} = f\left(M, \dfrac{N}{\sqrt{\theta}}\right)$	(Eq 4.31)	4.15
$\dfrac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(M, \dfrac{T_{N_x}}{\delta}\right)$	(Eq 4.32)	4.15
$T_{N_x} = T_G \cos \alpha_j - T_R$	(Eq 4.33)	4.15
$T_{N_x} = D$ (For small α_j, where $\cos \alpha_j \cong 1$)	(Eq 4.34)	4.15
$\dfrac{T_{N_x}}{\delta} = \dfrac{D}{\delta}$	(Eq 4.35)	4.15
$\dfrac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(M, \dfrac{D}{\delta}\right)$	(Eq 4.36)	4.15

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(M, \frac{W}{\delta}\right)$$ (Eq 4.37) 4.16

$$\theta = \frac{T_a}{T_{ssl}}$$ (Eq 4.38) 4.21

$$\theta_T = \frac{T_T}{T_{ssl}} = \frac{OAT}{T_{ssl}}$$ (Eq 4.39) 4.21

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(M, OAT\right)$$ (Eq 4.40) 4.22

$$T_T = T_a\left(1 + \frac{\gamma - 1}{2} M^2\right)$$ (Eq 4.41) 4.22

$$P_T = P_a\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$ (Eq 4.42) 4.22

$$\delta = \frac{P_a}{P_{ssl}}$$ (Eq 4.43) 4.23

$$\delta_T = \frac{P_T}{P_{ssl}}$$ (Eq 4.44) 4.23

$$\frac{\theta_T}{\theta} = \left(1 + 0.2\, M^2\right)$$ (Eq 4.45) 4.23

$$\frac{\delta_T}{\delta} = \left(1 + 0.2\, M^2\right)^{3.5}$$ (Eq 4.46) 4.23

$$\frac{\dot{W}_f}{\delta_T\sqrt{\theta_T}} = f\left(M, OAT\right)$$ (Eq 4.47) 4.23

$$TSFC = \frac{\dot{W}_f}{T_{N_x}}$$ (Eq 4.48) 4.25

$$T_{N_x} = D$$ (Eq 4.49) 4.26

$$\dot{W}_f \approx D$$ (Eq 4.50) 4.26

$$S.R. = \frac{nmi}{W_f}$$ (Eq 4.51) 4.28

$$S.R. = \frac{V_T}{\dot{W}_f}$$ (Eq 4.52) 4.28

$$S.E. = \frac{t}{W_{f_{Used}}}$$ (Eq 4.53) 4.28

$$S.E. = \frac{1}{\dot{W}_f}$$ (Eq 4.54) 4.29

$$Range = \left(S.R._{avg}\right)\left(Fuel\ Used\right)$$ (Eq 4.55) 4.31

$$nmi = \frac{nmi}{lb} \times lb$$ (Eq 4.56) 4.31

$$THP = \frac{T\ V_T}{550} = \frac{D\ V_T}{550}$$ (Eq 4.57) 4.33

$$THP = \frac{C_{D_P} \rho_a V_T^3 S}{1100} + \frac{W^2}{275\ \pi\ e\ AR\ S\ \rho_a\ V_T}$$ (Eq 4.58) 4.33

$$THP = THP_p + THP_i$$ (Eq 4.59) 4.34

$$D = \frac{C_D}{C_L} W$$ (Eq 4.60) 4.35

$$THP = \frac{C_D\ W\ V_T}{C_L\ 550}$$ (Eq 4.61) 4.35

$$L = W = C_L \frac{1}{2} \rho_a V_T^2 S$$ (Eq 4.62) 4.35

$$THP = \frac{\left(\frac{2}{S}\right)^{\frac{1}{2}} W^{\frac{3}{2}} C_D}{\rho_a^{\frac{1}{2}} C_L^{\frac{3}{2}} \ 550}$$ (Eq 4.63) 4.35

$$\rho_a = \rho_{ssl} \ \sigma$$ (Eq 4.64) 4.35

$$THP = \frac{\sqrt{2}\, W^{\frac{3}{2}} C_D}{\left(S \ \rho_{ssl}\right)^{\frac{1}{2}} \sqrt{\sigma} \ C_L^{\frac{3}{2}} \ 550}$$ (Eq 4.65) 4.35

$$THP_e = THP \sqrt{\sigma}$$ (Eq 4.66) 4.35

$$THP_e = \frac{\sqrt{2}\, W^{\frac{3}{2}} C_D}{\left(S \ \rho_{ssl}\right)^{\frac{1}{2}} C_L^{\frac{3}{2}} \ 550}$$ (Eq 4.67) 4.36

$$THP = K_1 V_T^3 + K_2 \ V_T^{-1}$$ (Eq 4.68) 4.37

$$3 \ K_1 V_T^2 - K_2 \ V_T^{-2} = 0$$ (Eq 4.69) 4.37

$$3 \ K_1 V_T^3 = K_2 \ V_T^{-1}$$ (Eq 4.70) 4.37

$$3 \ THP_p = THP_i$$ (Eq 4.71) 4.37

$$3 \ D_p = D_i$$ (Eq 4.72) 4.37

$$3 \ C_{D_p} = C_{D_i}$$ (Eq 4.73) 4.37

$$THP = K_o \frac{C_D}{C_L^{\frac{3}{2}}}$$

(Eq 4.74) 4.38

$$V_T = \frac{V_e}{\sqrt{\sigma}}$$

(Eq 4.75) 4.39

$$THP = \frac{THP_e}{\sqrt{\sigma}}$$

(Eq 4.76) 4.39

$$\sigma = \frac{\rho_a}{\rho_{ssl}}$$

(Eq 4.77) 4.40

$$THP_e \, V_e = \frac{C_{D_P} \rho_{ssl} V_e^4 S}{1100} + \frac{W^2}{275 \pi e \, AR \, S \, \rho_{ssl}}$$

(Eq 4.78) 4.40

$$\eta_P = \frac{THP}{SHP}$$

(Eq 4.79) 4.41

$$THP = \eta_P \, SHP$$

(Eq 4.80) 4.41

$$THPSFC = \frac{\dot{W}_f}{THP}$$

(Eq 4.81) 4.41

$$SHPSFC = \frac{\dot{W}_f}{SHP}$$

(Eq 4.82) 4.42

$$\dot{W}_f = \frac{THP}{\eta_P} \, SHPSFC$$

(Eq 4.83) 4.42

$$\left(\frac{W}{\delta}\right)_{Target} = \frac{W + W_f}{\delta}$$

(Eq 4.84) 4.47

$$\left(\frac{W}{\delta}\right)_{Target} = \frac{W_{aircraft} + W_f}{\delta}$$

(Eq 4.85) 4.48

$$\delta = \frac{W_{aircraft}}{\left(\frac{W}{\delta}\right)_{Target}} + \frac{1}{\left(\frac{W}{\delta}\right)_{Target}} W_f$$ (Eq 4.86) 4.48

$$OAT = T_a \left(1 + \frac{\gamma - 1}{2} K_T M^2\right)$$ (Eq 4.87) 4.48

$$H_{P_c} = H_{P_o} + \Delta H_{P_{ic}} + \Delta H_{pos}$$ (Eq 4.88) 4.59

$$T_a = T_o + \Delta T_{ic}$$ (Eq 4.89) 4.59

$$V_c = V_o + \Delta V_{ic} + \Delta V_{pos}$$ (Eq 4.90) 4.59

$$V_T = \frac{V_c}{\sqrt{\sigma}}$$ (Eq 4.91) 4.60

$$M = \frac{V_T}{a_{ssl}\sqrt{\theta}}$$ (Eq 4.92) 4.60

$$\dot{W}_{f_{ref}} = \frac{\dot{W}_f}{\delta\sqrt{\theta}}$$ (Eq 4.93) 4.60

$$W_{ref} = \frac{W}{\delta}$$ (Eq 4.94) 4.60

$$R_{Test} = \sum_{j=1}^{n} V_j \Delta t_j$$ (Eq 4.95) 4.63

$$R.F._{Test} = \frac{t_T}{\ln\left(\frac{W_1}{W_2}\right)}$$ (Eq 4.96) 4.63

$$R_{Std} = R.F._{Test} \ln\left(\frac{W_{Std_1}}{W_{Std_2}}\right)$$ (Eq 4.97) 4.63

$$M = f\left(V_c, H_{P_c}\right)$$ (Eq 4.98) 4.66

$$\delta = f\left(H_{P_c}\right)$$ (Eq 4.99) 4.67

$$\frac{W + W_f}{\delta} = \frac{W}{\delta}$$ (Eq 4.100) 4.67

$$\frac{W}{\delta}\,(\text{error}) = \frac{100\left[\frac{W}{\delta}\,(\text{test}) - \frac{W}{\delta}\,(\text{target})\right]}{\frac{W}{\delta}\,(\text{target})}$$ (Eq 4.101) 4.67

$$^{\circ}C = {}^{\circ}K - 273.15$$ (Eq 4.102) 4.67

$$OAT = f\left(T_a, M\right)$$ (Eq 4.103) 4.67

$$T_a = f\left(OAT, M\right)$$ (Eq 4.104) 4.67

$$S.R.\ \delta = \frac{661.483M}{\left(\frac{\dot{w}_f}{\delta\sqrt{\theta}}\right)}$$ (Eq 4.105) 4.68

$$V_e = \sqrt{\frac{2q}{\rho_{ssl}}} = \sqrt{\frac{\sigma 2q}{\rho_a}} = \sqrt{\sigma}\ V_T$$ (Eq 4.106) 4.73

$$SHP_e = SHP\sqrt{\sigma}$$ (Eq 4.107) 4.73

$$T_a = T_{a_{Std}} + \Delta T_a$$ (Eq 4.108) 4.78

$$S.R. = \frac{a_{ssl}\,M}{\left(\frac{\dot{W}_f}{\delta\sqrt{\theta}}\right)\delta}$$ (Eq 4.109) 4.78

$$V_{T_{Hot\ day}} = 661.483\ M\sqrt{\theta_{Std}}\left(\frac{\sqrt{\theta_{Hot\ day}}}{\sqrt{\theta_{Std}}}\right)$$ (Eq 4.110) 4.81

$$\dot{W}_{f_{\text{Hot day}}} = \dot{W}_{f_{\text{Std}}} \left(\frac{\sqrt{\theta_{\text{Hot day}}}}{\sqrt{\theta_{\text{Std}}}} \right) \qquad \text{(Eq 4.111)} \qquad 4.82$$

$$\frac{\dot{W}_f}{\sqrt{\theta}} = ¢ \qquad \text{(Eq 4.112)} \qquad 4.82$$

$$\text{Range} = \int_0^{W_{f_{\text{Used}}}} (\text{S.R.})\, dW_f \qquad \text{(Eq 4.113)} \qquad 4.86$$

$$\text{Range} = \int_{W_1}^{W_2} (\text{S.R.})\, dW \qquad \text{(Eq 4.114)} \qquad 4.87$$

$$\text{Range} = \int_{W_1}^{W_2} (\text{S.R. } \delta) \left(\frac{W}{\delta} \right) \frac{1}{W}\, dW \qquad \text{(Eq 4.115)} \qquad 4.87$$

$$\text{Range} = \int_{W_1}^{W_2} \left[\frac{661.483\, M}{\left(\frac{\dot{W}_f}{\delta \sqrt{\theta}} \right)} \frac{W}{\delta} \right] \frac{dW}{W} \qquad \text{(Eq 4.116)} \qquad 4.87$$

$$\frac{D}{\delta} = K_3 + K_4 \left(\frac{W}{\delta} \right)^2 \qquad \text{(Eq 4.117)} \qquad 4.88$$

$$\text{R.F.} = \left[(\text{S.R. } \delta) \frac{W}{\delta} \right] \qquad \text{(Eq 4.118)} \qquad 4.90$$

$$\text{R.F.} = \left[\frac{661.483\, M}{\left(\frac{\dot{W}_f}{\delta \sqrt{\theta}} \right)} \left(\frac{W}{\delta} \right) \right] \qquad \text{(Eq 4.119)} \qquad 4.90$$

$$\text{Range} = \int_{W_1}^{W_2} (\text{R.F.}) \frac{dW}{W} \qquad \text{(Eq 4.120)} \qquad 4.90$$

$$C_L = \frac{2\,W}{\gamma\,P_a\,M^2\,S} = \frac{2\,\frac{W}{\delta}}{\gamma\,M^2\,S\,P_{ssl}}$$ (Eq 4.121) 4.101

$$C_L = f\left(\frac{W}{\delta}, M^2\right)$$ (Eq 4.122) 4.101

$$\frac{\frac{W}{\delta}}{\frac{D}{\delta}} = \frac{W}{D} = \text{Constant}$$ (Eq 4.123) 4.101

$$\frac{661.483\,M}{\left(\frac{\dot{W}_f}{\delta\sqrt{\theta}}\right)} = \text{Constant}$$ (Eq 4.124) 4.102

$$\frac{TSFC}{\sqrt{\theta}} = \text{Constant}$$ (Eq 4.125) 4.102

$$\text{Range} = \text{R.F.}\ \ln\frac{W_1}{W_2}$$ (Eq 4.126) 4.105

CHAPTER 4

LEVEL FLIGHT PERFORMANCE

4.1 INTRODUCTION

This chapter examines the theory and flight tests to determine aircraft level flight performance. Of specific interest are range and endurance. Aerodynamic forces acting on the aircraft, lift and drag, and engine parameters are presented as functions of easily measured parameters. These engine and aerodynamic functions are combined to complete the analysis. The final result provides a method to determine engine and airplane performance characteristics with minimum flight testing. Only those aircraft and engine characteristics pertaining to level, unaccelerated flight are investigated. By definition, the aircraft is in level, unaccelerated flight when the sum of the forces and moments acting upon it equal zero. Therefore, lift equals aircraft weight, and the net thrust parallel flight path equals the aircraft drag.

4.2 PURPOSE OF TEST

The purpose of this test is to investigate aircraft performance characteristics in level flight to achieve the following objectives:

1. Determine significant performance parameters: maximum range and optimum range, maximum endurance and optimum endurance, and long range or ferry range.
2. Evaluate pertinent requirements of the specification.

4.3 THEORY

Aircraft level flight performance analysis is the process of determining standard day level flight characteristics from data obtained during nonstandard conditions. Until the advent of high speed aircraft and the accompanying compressibility effects, most flight test data were reduced by the density altitude method. The density altitude method can be used in the speed range where the assumption of constant drag for constant true speed and density altitude is valid. However, where effects of compressibility are not negligible this method results in erroneous standard day data. With jet powered aircraft came the necessity of standardizing data for constant compressibility conditions, thus avoiding compressibility corrections. This type of data reduction is the pressure altitude method. The pressure altitude method is based on the concept of maintaining a constant pressure altitude and indicated air speed, and correcting data only for temperature to obtain standard day performance. With identical test and standard day indicated airspeeds the test and standard day Mach numbers are the same.

Aerodynamic theory shows total drag is a function only of Mach number if weight and altitude are fixed. Reynold's number effects are generally ignored in flight test work. This is the basis for the simplicity and effectiveness of the pressure altitude method of flight test data reduction. Using this method, only a temperature correction to test day data is needed. Compressibility effects are automatically held constant.

4.3.1 DRAG

If a vehicle is to sustain flight, it must overcome the resistance to its motion through the air. The resistive force, acting in a direction opposing the direction of flight, is called aerodynamic drag.

A propulsion system, either propeller driven or jet, produces thrust to overcome the drag force on the vehicle. If the propulsive force or thrust is just equal and in opposite direction to the drag force on the vehicle in flight, the vehicle is in steady or equilibrium flight. If the thrust exceeds the drag, the vehicle accelerates.

The total drag on an aircraft is the sum of many component drags, such as the drag caused by the wings, fuselage, or tail. The aircraft manufacturer is interested in component drags when estimating the total drag of a proposed aircraft. The wind tunnel engineer is also interested in component drags, values he can measure experimentally on models in a wind tunnel. From these measurements, he attempts to predict the drag of a proposed or actual aircraft.

The flight test engineer is more interested in the total drag of an aircraft configuration, or in changes in total drag with changes in configuration. Total aircraft drag, rather than component drag, is the major consideration in aircraft performance determination. Total drag is determined normally from flight test measurements and is made up of many components.

4.3.1.1 SKIN FRICTION DRAG

Skin friction drag is caused by the viscosity of air flowing over the aircraft and is proportional to the shear stress on the aircraft surface caused by the airflow. Skin friction drag is created on all of the airframe surfaces exposed to the air stream such as wing, fuselage, and tail.

4.3.1.2 PRESSURE DRAG

Pressure drag arises because of the overall pressure distribution on an object. The difference between the forces caused by the high pressures on the forward portion and low pressures on the aft portion of the object is pressure drag. This drag is also know as wake or form drag because its magnitude is proportional to the size of the wake produced behind the object. Pressure drag always occurs since the total pressure is never completely recovered at the aft stagnation point, and there is always at least a small wake of separated flow behind any aerodynamic shape.

4.3.1.3 PROFILE DRAG

Profile drag is a measure of the resistance to flight caused by the viscous action of the air on the profile of the aircraft. The resistance to flight is the sum of skin friction and pressure drag. Profile drag is sometimes called boundary layer or viscous drag since neither skin friction nor pressure drag would occur if air were nonviscous. Equivalent flat plate area is accepted as a standard by which values of profile drag for aerodynamic shapes are compared.

4.3.1.4 INTERFERENCE DRAG

Interference drag is generated when several objects are placed in the same air stream creating eddy currents, turbulence, or restrictions to smooth flow. Any time two parts of an aircraft are joined or any object is placed on or in close proximity to an aircraft, interference drag is created. Fuselage-wing root junction and external stores hung on a wing are examples which produce this drag.

4.3.1.5 PARASITE DRAG

Parasite drag is the sum of profile and interference drag. It can be described as the drag which is not caused by lift or compressibility effects. Aircraft parasite drag is independent of angle of attack. Parasite drag is the total of each increment of drag caused by parts of the aircraft and its stores. A major source of parasite drag on reciprocating air cooled engine aircraft is called cooling drag where energy is lost as air is forced past cylinders of those piston powered propeller aircraft. The parasite drag equation is:

$$D_p = C_{D_p} qS \tag{Eq 4.1}$$

Where:

C_{D_p}	Parasite drag coefficient	
D_p	Parasite drag	lb
q	Dynamic pressure	psf
S	Wing area	ft^2.

4.3.1.6 INDUCED DRAG

The portion of the total drag force due to the production of lift is defined as induced drag. Since the function of lift is to overcome weight, induced drag can be correctly described as drag due to lift. Fortunately, the thrust or power required to overcome induced drag is not excessive, especially at high speeds.

As a wing produces lift, the vortices impart a final vertical velocity, V_v, to the air stream. Figure 4.1 depicts the airflow.

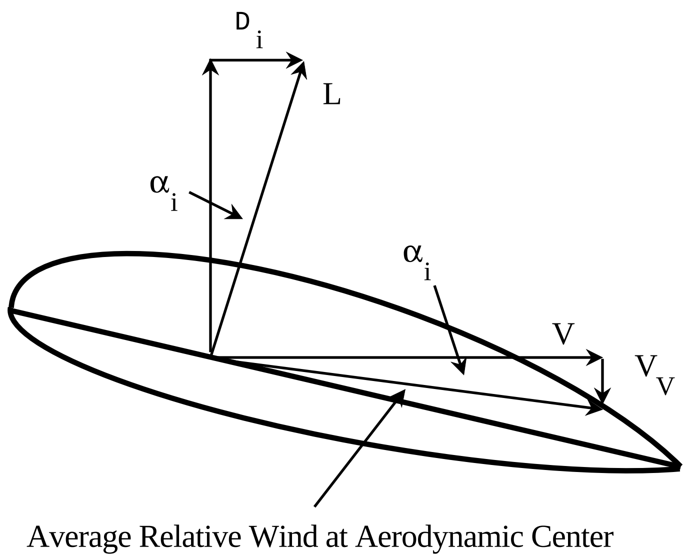

Figure 4.1
INDUCED FLOW FIELD

The term α_i is defined as the induced angle of attack in radians, and is the angle between the free stream relative wind and the average relative wind at the wing aerodynamic center. From the geometry of the lift and drag on an airfoil, the induced drag for small angles ($\sin \alpha_i \cong \alpha_i$) is:

$$D_i = L\,\alpha_i = C_L\, q\, S\, \alpha_i \tag{Eq 4.2}$$

$$L = C_L\, qS \tag{Eq 4.3}$$

$$D_i = C_{D_i}\, qS \tag{Eq 4.4}$$

Prandtl found if the wing has an elliptical planform, the induced angle of attack is constant across the span. In addition to the planform, the induced drag is dependent upon the aspect ratio (AR). The smaller the aspect ratio, the larger the induced angle of attack and the greater the downwash. Induced angle of attack (α_i) is a function of AR as follows:

$$\alpha_i = \frac{C_L}{\pi\, AR} \tag{Eq 4.5}$$

Substituting into Eq 4.2 and 4.4 for an elliptical airfoil:

$$C_{D_i} = \frac{C_L^2}{\pi \, AR} \qquad \text{(Eq 4.6)}$$

An induced drag coefficient can be defined for any given planform by inserting a constant, Oswald's efficiency factor, (e). The constant can be determined for any given aircraft configuration from flight test data and is used in the total drag coefficient equation for any wing planform or for an entire aircraft. Eq 4.6 becomes:

$$C_{D_i} = \frac{C_L^2}{\pi \, e \, AR} \qquad \text{(Eq 4.7)}$$

Where:

α_i	Induced angle of attack	deg
AR	Aspect ratio	
C_{D_i}	Induced drag coefficient	
C_L	Lift coefficient	
D_i	Induced drag	lb
e	Oswald's efficiency factor	
L	Lift	lb
π	Constant	
q	Dynamic pressure	psf
S	Wing area	ft^2.

4.3.1.7 WAVE DRAG

Wave drag, often called compressibility or Mach drag, is the drag resulting when flow over the surfaces of an aircraft exceeds Mach 1.0. Supersonic flow over aircraft surfaces results in the formation of shock waves, causing a sizeable increase in drag due to the large pressure changes across the shock. Behind the shock wave, the flow field operates in an adverse pressure gradient due to the large increase in static pressure as the velocity is slowed to a lower supersonic or subsonic value. The net drag due to this higher pressure behind a shock wave is the wave drag.

4.3.1.8 RAM DRAG

Ram drag is drag due to ram compression aft of the compressor prior to the combustion section in the diffuser of a turbojet or turbofan engine. This term is widely used in propulsion. Net thrust is usually defined as gross thrust minus ram drag.

4.3.1.9 TRIM DRAG

Trim drag is an additional drag force resulting from the use of the horizontal tail in trimming the aircraft. The tail may carry a download for which the wing must provide additional lift. The increase in angle of attack required for this additional lift also increases the drag and reduces the critical Mach. The trim drag is significant at high altitudes particularly in designs with a short tail moment arm. The download requirements can be reduced by reducing the static margin (moving the CG aft). Canard configurations produce an upload for trim which is equivalent to providing negative trim drag.

4.3.1.10 TOTAL DRAG

For flight test, a detailed breakdown of all drag components is not necessary. Conventionally, all drag which is not a function of lift is called parasite drag, and all drag which is a function of lift is called induced drag. If the aircraft velocity is greater than the critical Mach, wave drag accounts for the losses due to shock waves. Total drag may be written as the sum of the major component drags or sum of the drag coefficients using definitions similar to Eq 4.1.

$$D = D_P + D_i + D_M \tag{Eq 4.8}$$

$$C_D = C_{D_P} + C_{D_i} + C_{D_M} \tag{Eq 4.9}$$

Where:

C_D	Drag coefficient
C_{D_i}	Induced drag coefficient
C_{D_M}	Mach drag coefficient
C_{D_p}	Parasite drag coefficient

D	Drag	lb
D_i	Induced drag	lb
D_M	Mach drag	lb
D_p	Parasite drag	lb.

4.3.1.11 LOW SPEED DRAG CHARACTERISTICS

An equation for low speed total drag, ignoring Mach effects, is presented below using Eq 4.8 and assuming a parabolic drag polar (C_{D_i} is proportional to $C_L{}^2$).

$$D = C_D \, qS \tag{Eq 4.10}$$

$$C_D = C_{D_p} + \frac{C_L^2}{\pi \, e \, AR} \tag{Eq 4.11}$$

$$D = C_{D_P} \, q \, S + \frac{C_L^2}{\pi \, e \, AR} \, qS \tag{Eq 4.12}$$

An expression for C_L is developed to quantify the drag. The forces acting on an airplane in level flight are given in figure 4.2.

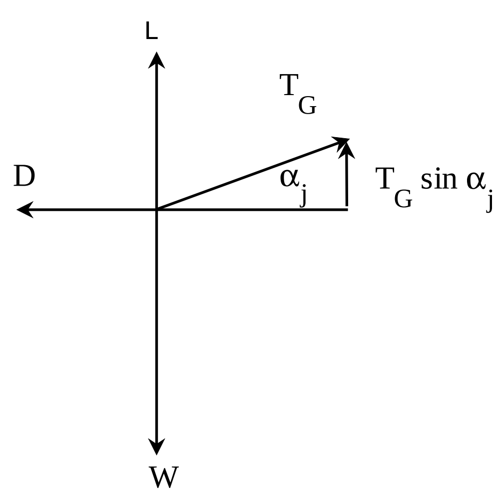

Figure 4.2
FORCES IN LEVEL FLIGHT

To maintain level unaccelerated flight the sum of the forces in the z axis equals zero:

$$L - W + T_G \sin \alpha_j = 0 \tag{Eq 4.13}$$

$$L = W - T_G \sin \alpha_j \tag{Eq 4.14}$$

The lift coefficient is defined and substituted into Eq 4.12.

$$C_L = \frac{L}{qS} \tag{Eq 4.15}$$

$$C_L = \frac{W - T_G \sin \alpha_j}{qS} \tag{Eq 4.16}$$

$$D = C_{D_P} qS + \frac{\left(W - T_G \sin \alpha_j\right)^2}{\pi\, e\, AR\, qS} \tag{Eq 4.17}$$

To simplify the above equation, assume weight (W) is much larger than the vertical thrust component ($T \sin \alpha_j$). This is a valid assumption for airplanes in cruise where α_j is small. At low speed, large α_j and high thrust setting may introduce sizeable error. Eq 4.17 reduces to:

$$D = C_{D_P} qS + \frac{W^2}{\pi\, e\, AR\, qS} \tag{Eq 4.18}$$

Dynamic pressure (q) can be expressed in any of the following forms:

$$q = \frac{1}{2} \rho_{ssl} V_e^2 \tag{Eq 4.19}$$

$$q = \frac{1}{2} \rho_a V_T^2 \tag{Eq 4.20}$$

$$q = \frac{1}{2} \gamma P_a M^2 \qquad \text{(Eq 4.21)}$$

The level flight drag Eq 4.18 can then become:

$$D = \frac{C_{D_p} \rho_{ssl} V_e^2 S}{2} + \frac{2 W^2}{\pi e AR S \rho_{ssl} V_e^2} \qquad \text{(Eq 4.22)}$$

$$D = \frac{C_{D_p} \rho_a V_T^2 S}{2} + \frac{2 W^2}{\pi e AR S \rho_a V_T^2} \qquad \text{(Eq 4.23)}$$

$$D = \frac{C_{D_p} \gamma P_a M^2 S}{2} + \frac{2W^2}{\pi e AR S \gamma P_a M^2} \qquad \text{(Eq 4.24)}$$

Where:

α_j	Thrust angle	deg
AR	Aspect ratio	
C_D	Drag coefficient	
C_{D_p}	Parasite drag coefficient	
C_L	Lift coefficient	
D	Drag	lb
e	Oswald's efficiency factor	
γ	Ratio of specific heats	
L	Lift	lb
M	Mach number	
π	Constant	
P_a	Ambient pressure	psf
q	Dynamic Pressure	psf
ρ_a	Ambient air density	slugs/ft^3
ρ_{ssl}	Standard sea level air density	0.0023769 slugs/ft^3
S	Wing area	ft^2

T_G	Gross thrust	lb
V_e	Equivalent airspeed	kn
V_T	True airspeed	kn
W	Weight	lb.

A sketch of the drag equations in general form is shown in figure 4.3.

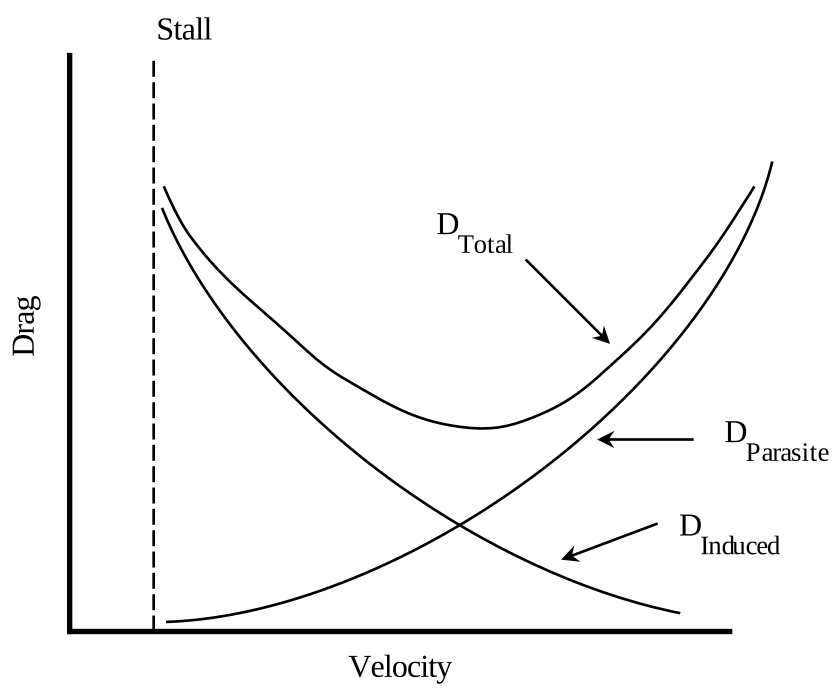

Figure 4.3
DRAG CURVES

The general features of the total drag curve are:

1. Positive sloped segment often called the front side.
2. Negative sloped segment called the back side.
3. Minimum drag point called the bucket.

4.3.1.12 HIGH MACH DRAG

At high transonic Mach numbers, the drag polar parameters (C_{D_p} and e) start to vary. Eq 4.24 is the level flight drag equation expressed as a function of Mach. For the low speed case C_{D_p} and e are constant. This simplification is not possible in the high Mach case since C_{D_p} and e vary. Figure 4.4 is a sketch showing typical C_{D_p} and e variations with Mach.

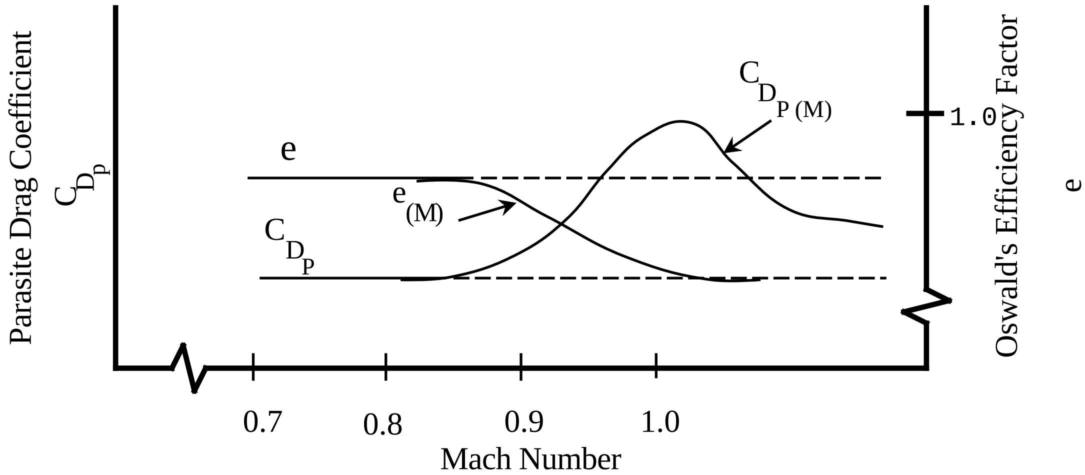

Figure 4.4
MACH EFFECT ON DRAG

A generalized form of drag equation at high Mach is:

$$D = \frac{C_{D_{p\,(M)}} \gamma P_a M^2 S}{2} + \frac{2W^2}{\pi e_{(M)} AR\, S\, \gamma P_a M^2} \qquad \text{(Eq 4.25)}$$

Mach drag is accounted for in the parasite and induced drag terms. There are three independent variables (M, W, P_a) in Eq 4.25. To document the airplane, drag in level flight at various Mach numbers would be measured. At each altitude the weight would be varied over its allowable range. This approach would be a major undertaking. To simplify the analysis, the independent variables are reduced to two by multiplying both sides of Eq 4.25 by $\frac{P_{ssl}}{P_a}$ and the induced drag term by $\frac{P_{ssl}}{P_{ssl}}$. Substituting $\delta = \frac{P_a}{P_{ssl}}$ and simplifying:

$$\frac{D}{\delta} = \frac{C_{D_{P(M)}} \gamma P_{ssl} M^2 S}{2} + \frac{2\left(W/\delta\right)^2}{\pi e_{(M)} AR\, S\, \gamma P_{ssl} M^2} \qquad \text{(Eq 4.26)}$$

Eq 4.26 in functional notation is:

$$\frac{D}{\delta} = f\left(M, \frac{W}{\delta}\right) \qquad \text{(Eq 4.27)}$$

Where:

AR	Aspect ratio	
$C_{D_{p\,(M)}}$	Parasite drag coefficient at high Mach	
D	Drag	lb
δ	Pressure ratio	
$e_{(M)}$	Oswald's efficiency factor at high Mach	
γ	Ratio of specific heats	
M	Mach number	
π	Constant	
P_a	Ambient pressure	psf
P_{ssl}	Standard sea level pressure	2116.217 psf
S	Wing area	ft^2
W	Weight	lb.

This grouping of terms allows drag data to be correlated and corrected, or referred to other weights and altitudes. Documenting the airplane operating envelope (M, W, H_P) can be efficiently accomplished. Graphically this relationship looks like figure 4.5.

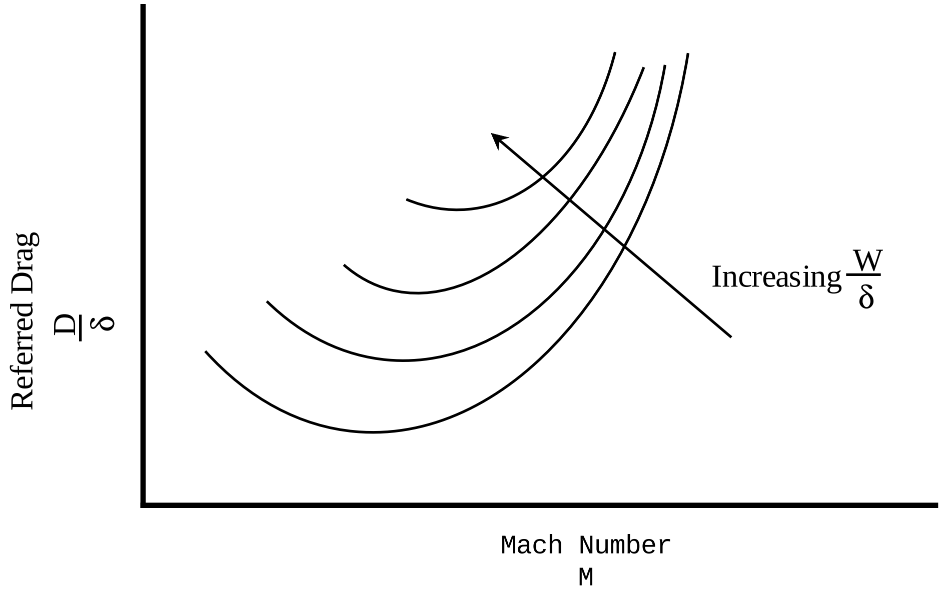

Figure 4.5
LEVEL FLIGHT DRAG

4.3.2 JET THRUST REQUIRED

Fuel flow, $\dot{W}_f$, is a function of both fluid properties and engine variables. Dimensional analysis shows:

$$\dot{W}_f = f\left(P, \rho, \mu, V, L, N\right) \tag{Eq 4.28}$$

A parameter called referred fuel flow, $\dfrac{\dot{W}_f}{\delta\sqrt{\theta}}$, is functionally expressed as follows:

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(M, \frac{N}{\sqrt{\theta}}, R_e\right) \tag{Eq 4.29}$$

Referred fuel flow and referred engine speed, $\dfrac{N}{\sqrt{\theta}}$, are not the only referred engine parameters. The complete list includes six others. These parameters can be derived mathematically from the Buckingham Pi Theorem, or related to the physical phenomena occurring in the engine. Neglecting Reynold's number, Eq 4.29 becomes:

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(M, \frac{N}{\sqrt{\theta}}\right) \tag{Eq 4.30}$$

Referred net thrust parallel flight path can be defined as $\frac{T_{N_x}}{\delta}$ and is a function of the same parameters as fuel flow:

$$\frac{T_{N_x}}{\delta} = f\left(M, \frac{N}{\sqrt{\theta}}\right) \tag{Eq 4.31}$$

Referred fuel flow can be expressed functionally:

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(M, \frac{T_{N_x}}{\delta}\right) \tag{Eq 4.32}$$

In unaccelerated flight, net thrust parallel flight path is equal to net or total drag:

$$T_{N_x} = T_G \cos \alpha_j - T_R \tag{Eq 4.33}$$

$$T_{N_x} = D \text{ (For small } \alpha_j\text{, where } \cos \alpha_j \cong 1\text{)} \tag{Eq 4.34}$$

$$\frac{T_{N_x}}{\delta} = \frac{D}{\delta} \tag{Eq 4.35}$$

Eq 4.32 becomes:

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(M, \frac{D}{\delta}\right) \tag{Eq 4.36}$$

From Eq 4.27, referred fuel flow and referred gross weight are functionally related as follows:

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(M, \frac{W}{\delta}\right) \qquad \text{(Eq 4.37)}$$

Where:

α_j	Thrust angle	deg
D	Drag	lb
δ	Pressure ratio	
H_P	Pressure altitude	ft
L	Lift	lb
M	Mach number	
μ	Viscosity	lb-s/ft^2
N	Engine speed	RPM
P	Pressure	psf
θ	Temperature ratio	
ρ	Air density	slugs/ft^3
R_e	Reynold's number	
T_G	Gross thrust	lb
T_{N_x}	Net thrust parallel flight path	lb
T_R	Ram drag	lb
V	Velocity	kn
W	Weight	lb
$\dot{W}_f$	Fuel flow	lb/h.

The relationship expressed in Eq 4.37 is presented in figure 4.6.

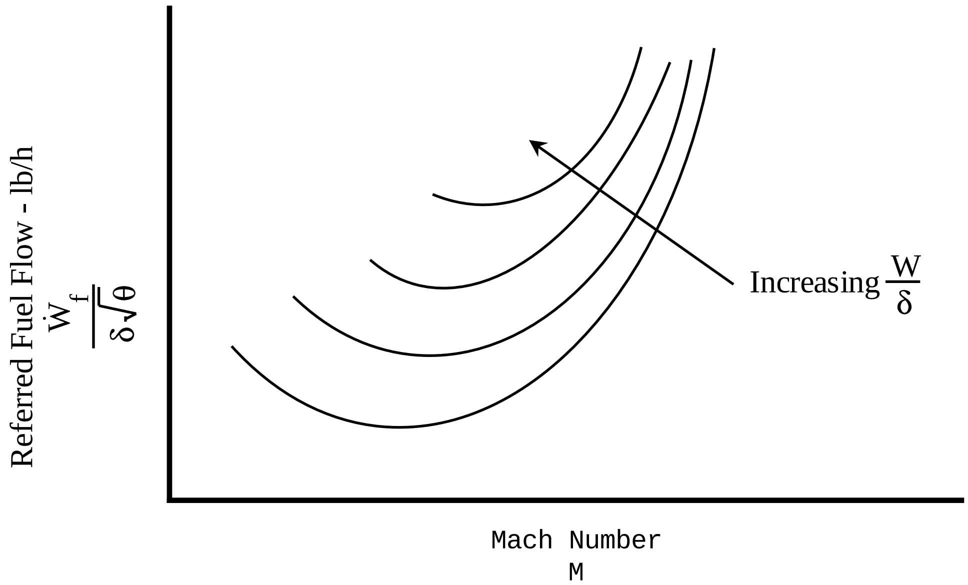

Figure 4.6
REFERRED FUEL FLOW

All variables affecting range and endurance are contained in figure 4.6, including velocity, fuel flow, gross weight, altitude, and ambient temperature. Range and endurance can be determined without directly measuring thrust or drag.

4.3.3 JET THRUST AVAILABLE

In level flight, thrust available determines maximum level flight airspeed, V_H, as shown in figure 4.7. For a turbojet, V_H takes the following forms:

M_{mrt}	Mach number at military rated thrust
M_{max}	Mach number at maximum thrust

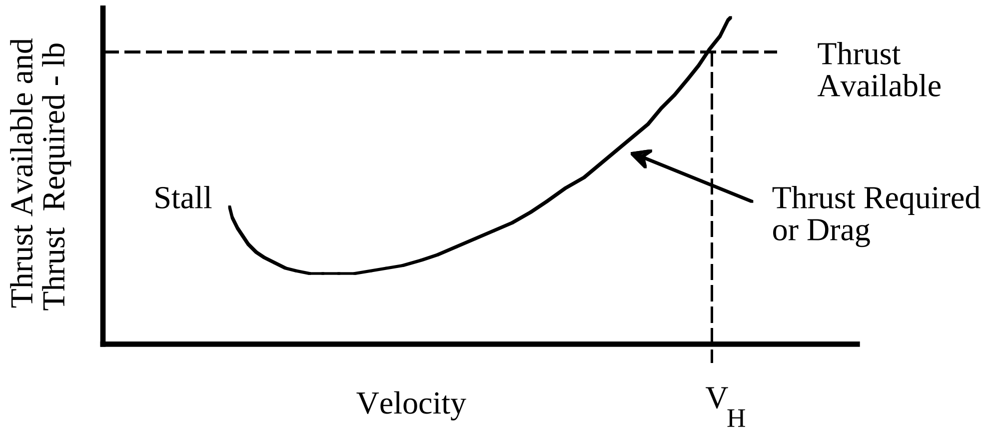

Figure 4.7
MAXIMUM LEVEL FLIGHT AIRSPEED

In performance testing, the aircraft is flown to a stabilized maximum level flight airspeed (V_H) where thrust equals drag. A second condition satisfied at this point is fuel flow required equals fuel flow available as depicted in figure 4.8.

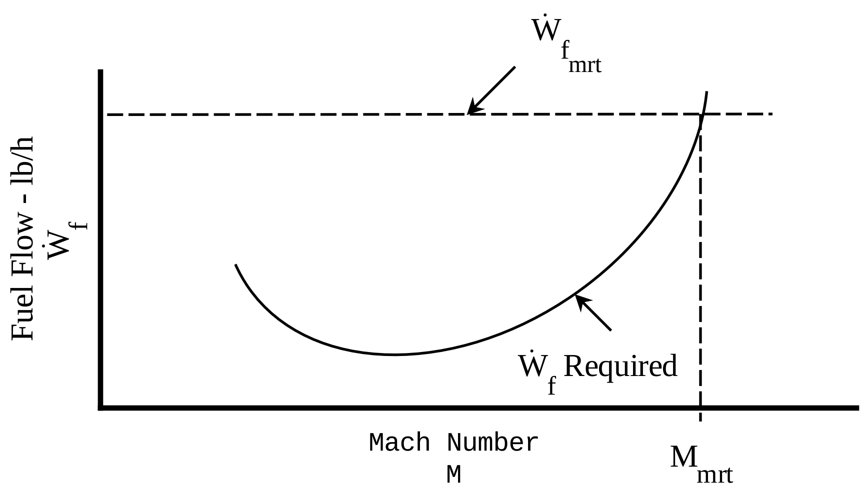

Figure 4.8
FUEL FLOW VERSUS MACH NUMBER

4.3.3.1 FUEL FLOW CORRECTION

Thrust required depends on, gross weight, pressure altitude, and Mach number as far as drag is concerned. Thrust available depends on power setting, pressure altitude, Mach, and ambient temperature. Test day M_{mrt} occurs at the point where test condition fuel flow required equals the test condition fuel flow available at military power. Evaluating the aircraft to other conditions requires referring the test conditions to standard conditions and then adjusting M_{mrt} (test) to M_{mrt} (standard). The technique determines M_{mrt} for specified conditions by determining the intersection of referred fuel flow required and referred fuel flow available. These parameters are calculated separately and combined to find the intersection.

4.3.3.1.1 REFERRED FUEL FLOW REQUIRED

The test results for a given configuration, gross weight, and pressure altitude can be plotted as in figure 4.6. From the figure, fuel flow is independent of ambient temperature. The same curve applies for cold day, standard day, or hot day.

4.3.3.1.2 REFERRED FUEL FLOW AVAILABLE

Referred fuel flow available is not independent of ambient temperature. From engine propulsion studies, variables affecting referred fuel flow available are: power setting, pressure altitude, Mach number, and ambient temperature. The temperature effect is illustrated by 5 power available curves presented in figure 4.9. M_{mrt} increases as the ambient temperature decreases.

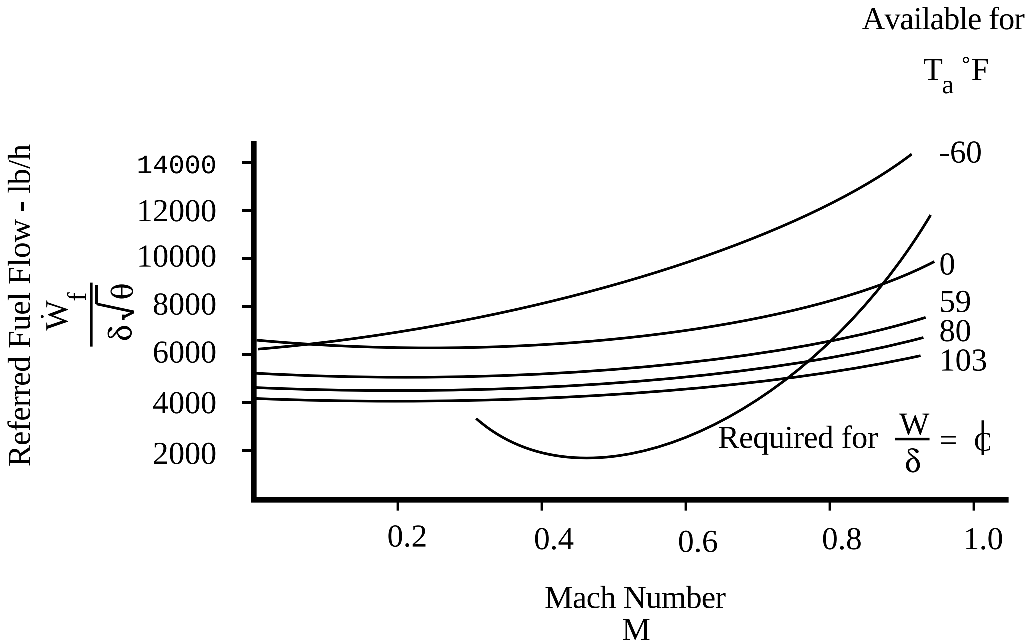

Figure 4.9
REFERRED FUEL FLOW AVAILABLE

For a fixed geometry turbojet and ignoring Reynold's number, referred fuel flow depends upon Mach and $\frac{N}{\sqrt{\theta}}$. The engine fuel control schedules engine speed, N, as a function of the inlet total temperature (at engine compressor face),T_{T_2}, in a manner similar to figure 4.10.

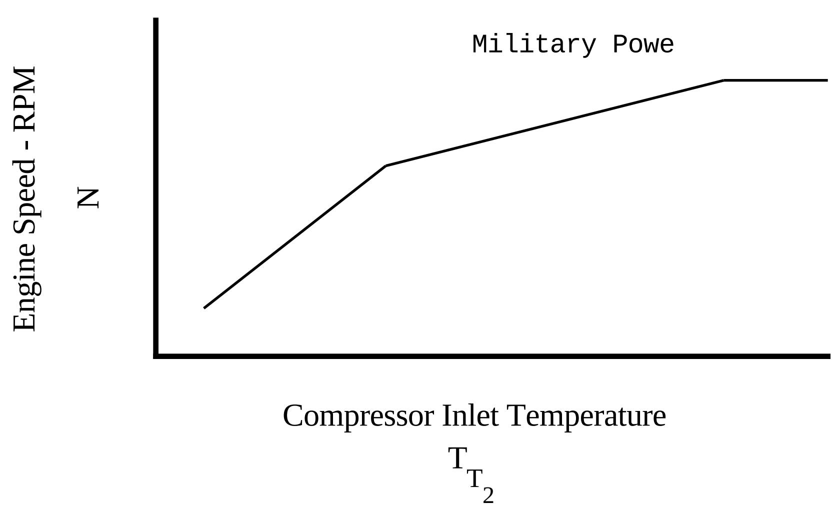

Figure 4.10
ENGINE SCHEDULING

The temperature at the compressor face is equal to the ambient temperature or outside air temperature. The temperature ratio, θ, and total temperature ratio, θ_T, are defined as:

$$\theta = \frac{T_a}{T_{ssl}} \tag{Eq 4.38}$$

$$\theta_T = \frac{T_T}{T_{ssl}} = \frac{OAT}{T_{ssl}} \tag{Eq 4.39}$$

The outside air temperature can be related to the total temperature ratio and referred engine speed as shown in figure 4.11.

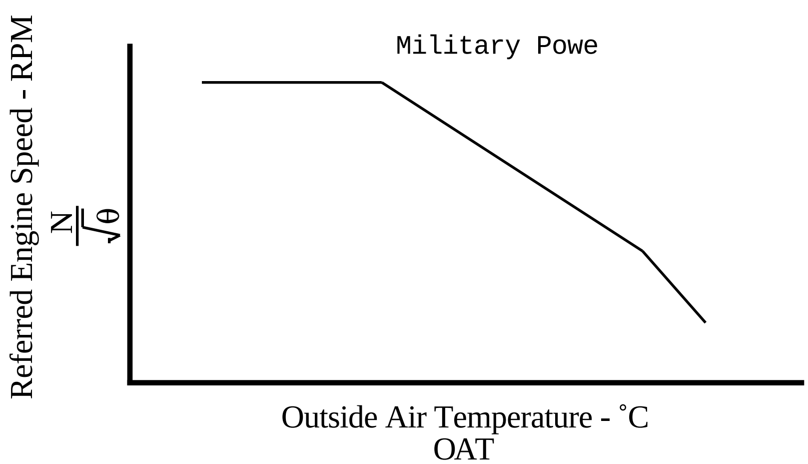

Figure 4.11
REFERRED ENGINE SCHEDULING

The referred engine speed is fairly linear over certain ranges of OAT with break points in the schedules. These break points occur because the engine switches modes of control as the OAT increases. At low temperatures it may be running to a maximum airflow schedule, and at hotter temperatures it runs to maximum turbine temperature or maximum physical speed. To calculate M_{mrt} for one power setting Eq 4.30 can be rewritten as:

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(M, OAT\right) \qquad \text{(Eq 4.40)}$$

The function plotted over the range of flight test data appears as in figure 4.12.

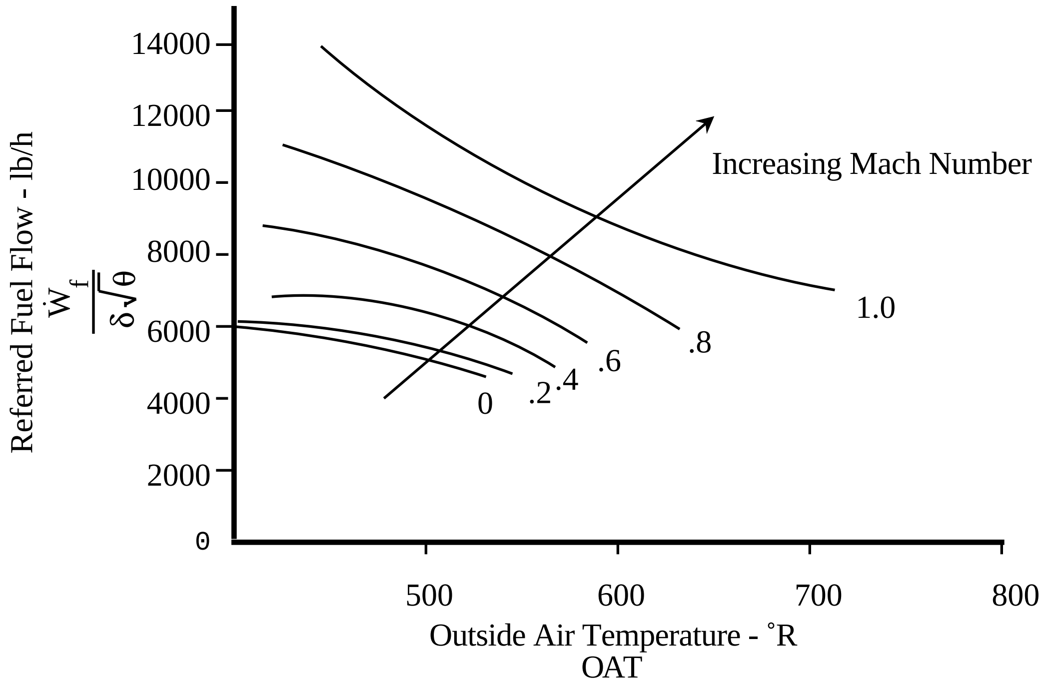

Figure 4.12
REFERRED FUEL FLOW AND OAT

Figure 4.12 can be simplified for analysis by defining a referred fuel flow which accounts for Mach effects. From total properties:

$$T_T = T_a\left(1 + \frac{\gamma - 1}{2}\ M^2\right) \qquad \text{(Eq 4.41)}$$

$$P_T = P_a\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma - 1}} \qquad \text{(Eq 4.42)}$$

$$\theta = \frac{T_a}{T_{ssl}} \qquad \text{(Eq 4.38)}$$

$$\delta = \frac{P_a}{P_{ssl}} \tag{Eq 4.43}$$

$$\theta_T = \frac{T_T}{T_{ssl}} = \frac{OAT}{T_{ssl}} \tag{Eq 4.39}$$

$$\delta_T = \frac{P_T}{P_{ssl}} \tag{Eq 4.44}$$

Letting γ equal 1.4 for the temperatures normally encountered, Eq 4.41 and 4.42 simplify to:

$$\frac{\theta_T}{\theta} = \left(1 + 0.2\ M^2\right) \tag{Eq 4.45}$$

$$\frac{\delta_T}{\delta} = \left(1 + 0.2\ M^2\right)^{3.5} \tag{Eq 4.46}$$

Since $\theta_T = f\ (\theta, M)$ and $\delta_T = f\ (\delta, M)$ a parameter is defined as fuel flow referred to total conditions and figure 4.12 is replotted as figure 4.13.

$$\frac{\dot{W}_f}{\delta_T \sqrt{\theta_T}} = f\left(M, OAT\right) \tag{Eq 4.47}$$

Where:

δ	Pressure ratio	
δ_T	Total pressure ratio	
γ	Ratio of specific heats	
M	Mach number	
OAT	Outside air temperature	˚C or ˚K
P_a	Ambient pressure	psf
P_{ssl}	Standard sea level pressure	2116.217 psf
P_T	Total pressure	psf
θ	Temperature ratio	

θ_T	Total temperature ratio	
T_a	Ambient temperature	˚C or ˚K
T_{ssl}	Standard sea level temperature	15˚C, 288.15˚K
T_T	Total temperature	˚C,˚K
$\dot{W}_f$	Fuel flow	lb/h.

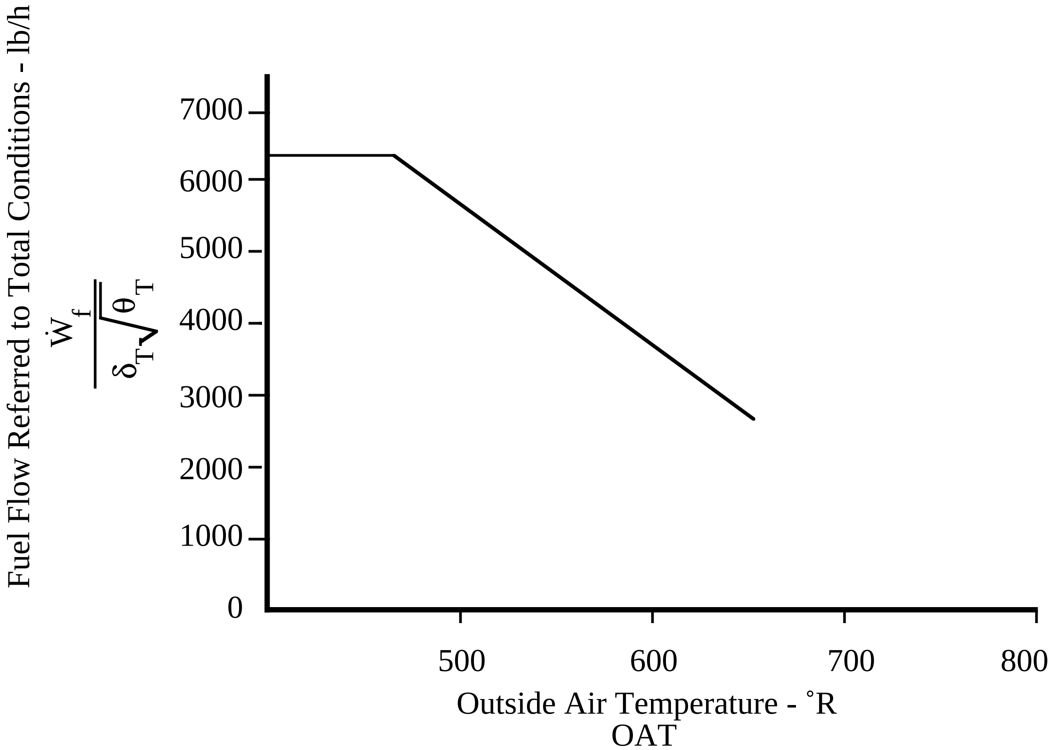

Figure 4.13
REFERRED FUEL FLOW AT TOTAL CONDITIONS

In figure 4.13, fuel flow is not dependent on Mach. The data for figure 4.13 can come from any military power condition, regardless of pressure altitude or ambient temperature. The aircraft need not be in equilibrium, allowing the data points to be taken during military power climbs and accelerations. A sufficient number of points from the W/δ flights can be obtained by including a M_{mrt} point for each W/δ. After all of the W/δ flights are complete, a curve similar to figure 4.14 is plotted. The curve should be fairly linear and can be extrapolated over a small range of OAT. Using figures 4.6 and 4.14, fuel flow can be unreferred to calculate M_{mrt} for any condition.

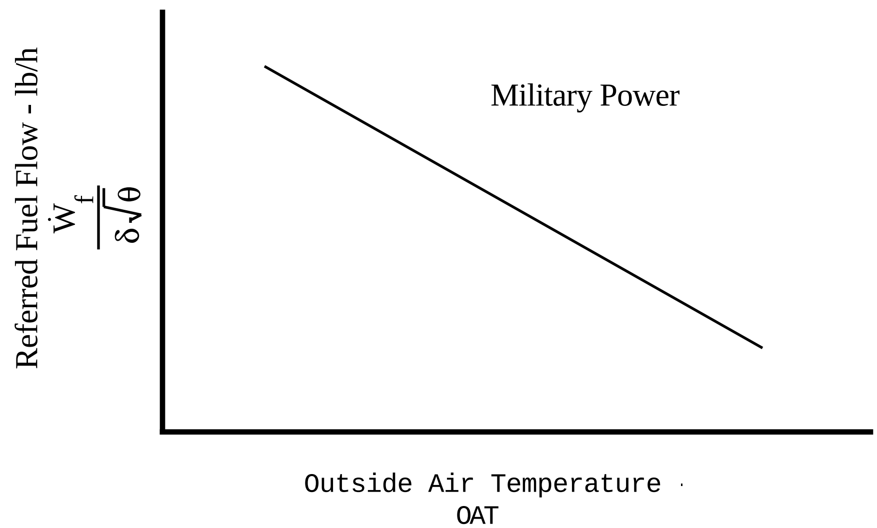

Figure 4.14
LINEARIZED REFERRED FUEL FLOW AVAILABLE

4.3.4 JET RANGE AND ENDURANCE

The ability of an airplane to convert fuel energy into flying distance is a high priority performance item. Efficient range characteristics are specified in either of two general forms:

1. To extract the maximum flying distance from a given fuel load.
2. To fly a specified distance with minimum expenditure of fuel.

The common denominator for each is the specific range (S.R.) or nautical miles per pound of fuel. For a jet, fuel flow is approximately proportional to the thrust produced. Thrust Specific Fuel Consumption (TSFC) can be defined as the ratio of fuel flow to net thrust parallel flight path and is shown in figure 4.15.

$$\text{TSFC} = \frac{\dot{W}_f}{T_{N_x}} \qquad \text{(Eq 4.48)}$$

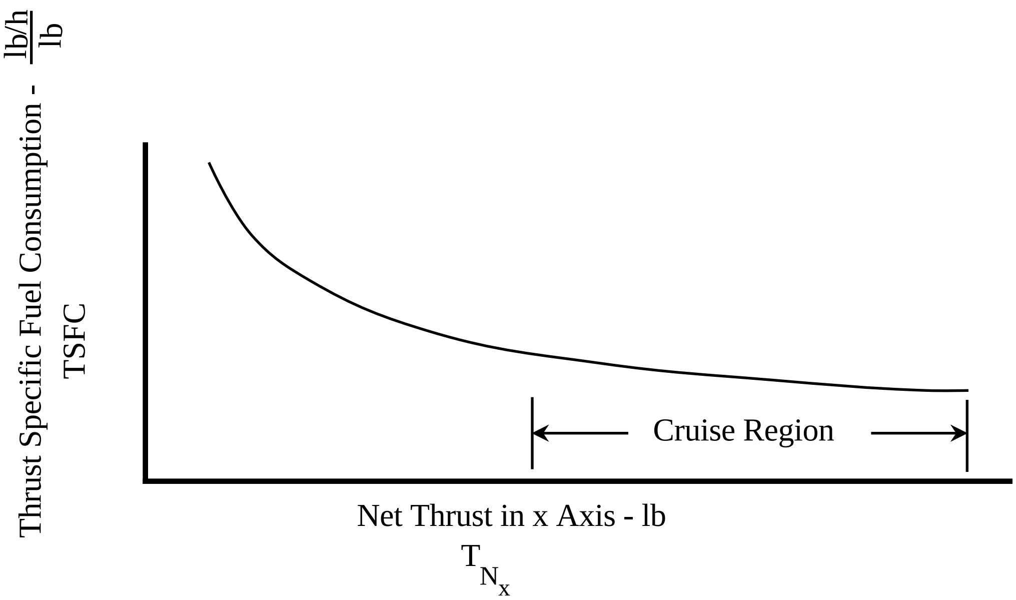

Figure 4.15
THRUST SPECIFIC FUEL CONSUMPTION

In the cruise region, TSFC is essentially constant. Therefore, the drag curve and the fuel flow required curve are similar. This can be shown by letting net thrust parallel flight path equal drag then fuel flow would be proportional to drag.

$$T_{N_x} = D \tag{Eq 4.49}$$

$$\dot{W}_f \approx D \tag{Eq 4.50}$$

A graph of fuel flow and drag characteristics, assuming constant TSFC, is shown in figure 4.16.

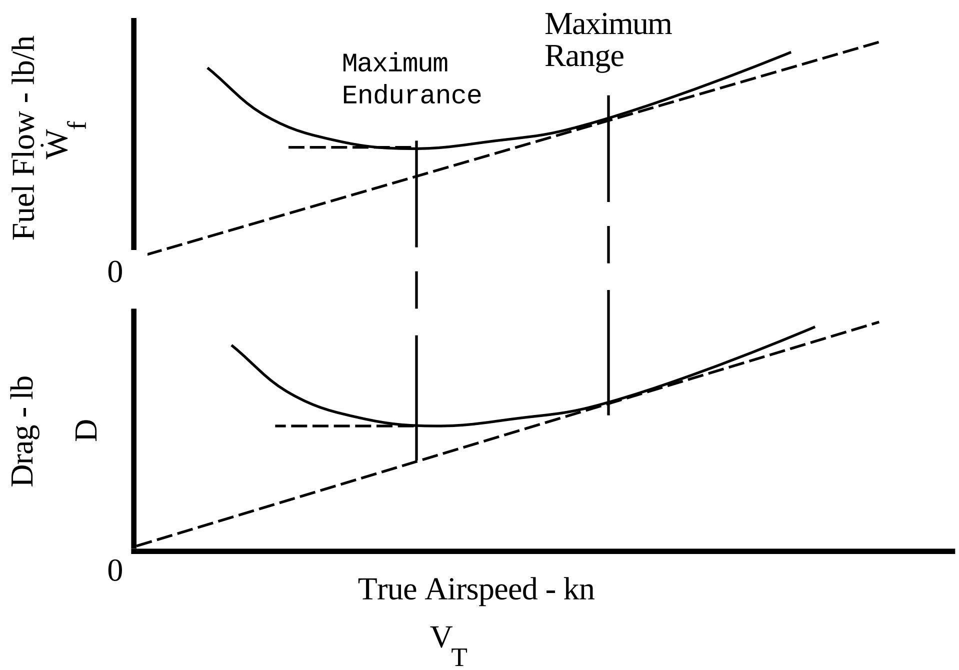

Figure 4.16
JET RANGE AND ENDURANCE

Maximum range occurs at the tangent to the drag curve, and maximum endurance occurs at minimum drag.

The thrust specific fuel consumption is not constant but is a function of many engine propulsion variables. In general, engine efficiency increases with increasing engine speed and decreasing inlet temperature. Specific range increases with increasing altitude. Likewise, endurance is not constant with increasing altitude. Minimum fuel flow decreases with increasing altitude until the optimum altitude is reached, then begins to increase again. The decrease in fuel flow is due to increasing engine efficiency as speed increases and temperature decreases. Eventually, decreased Reynold's number and increased Mach negate the positive effects of increasing altitude and an optimum altitude is reached.

Range must be distinguished from endurance. Range involves flying distance while endurance involves flying time. The appropriate definition of the latter is specific endurance (S.E). Endurance equates to flying the maximum amount of time for the least amount of fuel.

To determine aircraft range and endurance, the thrust or power required must be put in terms of actual aircraft fuel flow required. Figure 4.17 depicts where the maximum range and endurance occur.

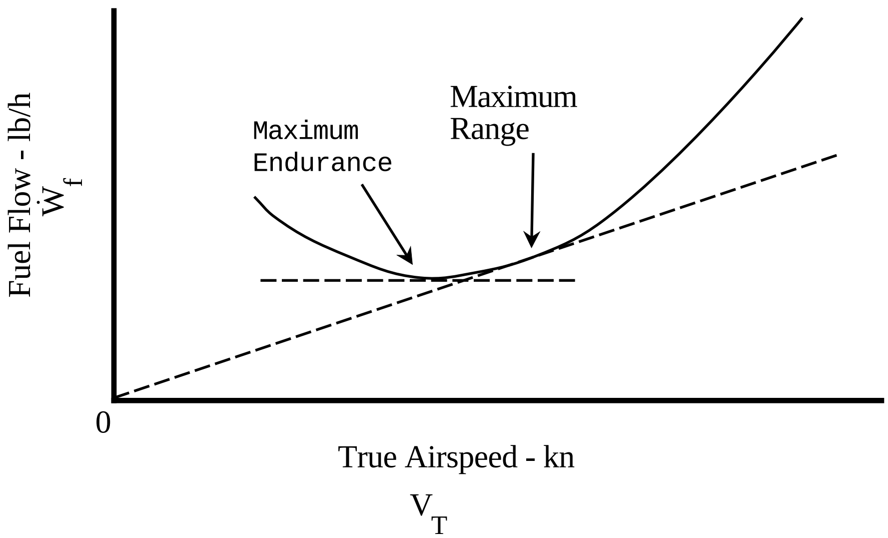

Figure 4.17
FUEL FLOW REQUIRED

Specific Range can be defined and calculated by dividing true airspeed by the actual fuel flow.

$$S.R. = \frac{nmi}{W_f} \qquad \text{(Eq 4.51)}$$

$$S.R. = \frac{V_T}{\dot{W}_f} \qquad \text{(Eq 4.52)}$$

Maximum range airspeed is the speed at the tangent from the origin to the fuel flow curve (Figure 4.17). Maximum endurance airspeed is located where fuel flow is minimum (Figure 4.17). Specific endurance is calculated by dividing flight time by fuel used such that:

$$S.E. = \frac{t}{W_{f_{Used}}} \qquad \text{(Eq 4.53)}$$

$$S.E. = \frac{1}{\dot{W}_f} \qquad \text{(Eq 4.54)}$$

Where:

D	Drag	lb
nmi	Nautical miles	
S.E.	Specific endurance	h/lb
S.R.	Specific range	nmi/lb
t	Time	s
T_{N_x}	Net thrust parallel flight path	lb
TSFC	Thrust specific fuel consumption	$\frac{lb/h}{lb}$
V_T	True airspeed	kn
$W_{f_{Used}}$	Fuel used	lb
$\dot{W}_f$	Fuel flow	lb/h.

An analysis of range performance can be obtained by plotting specific range versus velocity as shown in figure 4.18.

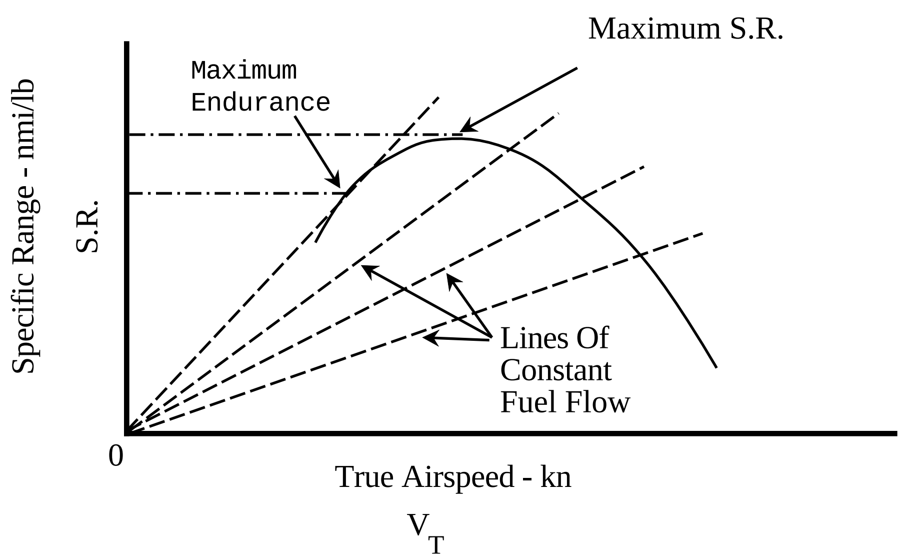

Figure 4.18
RANGE AND ENDURANCE

The maximum specific range is at the peak of the curve while maximum endurance is the tangent to the curve through the origin. Long range cruise operation is conducted generally at some airspeed slightly higher than the airspeed for maximum S.R. which does not significantly reduce range but does shorten enroute time.

The curves of specific range versus velocity are affected by three principal variables: airplane weight, the external aerodynamic configuration of the airplane, and altitude. These curves are the source of range and endurance operating data and are included in the operator's flight handbook.

4.3.4.1 WEIGHT EFFECTS

Total range is dependent on both the fuel available and the specific range. A typical variation of specific range with weight for a particular cruise operation is given in figure 4.19.

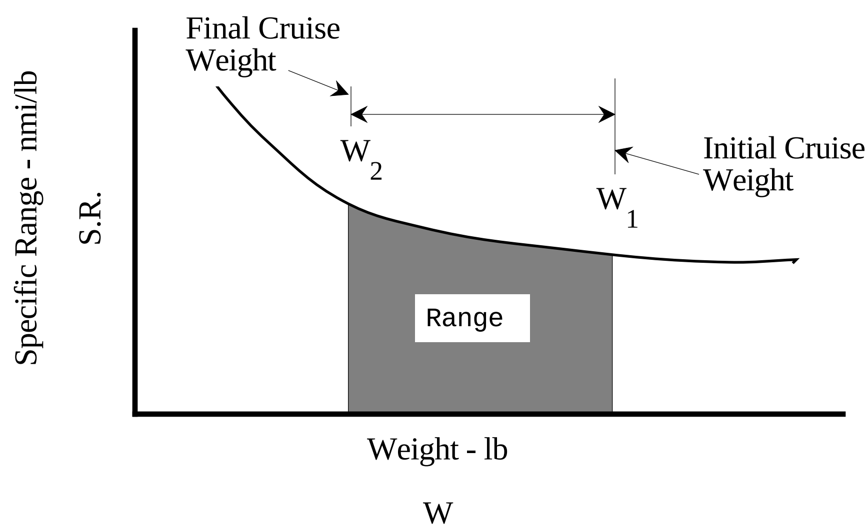

Figure 4.19
SPECIFIC RANGE VARIATION WITH WEIGHT

Range obtained by the expenditure of fuel can be related to the crosshatched area between the weights at the beginning and end of cruise. The actual range is computed by multiplying the average specific range by the pounds of fuel expended.

$$\text{Range} = \left(\text{S.R.}_{avg}\right)\left(\text{Fuel Used}\right) \qquad \text{(Eq 4.55)}$$

$$nmi = \frac{nmi}{lb} \times lb \tag{Eq 4.56}$$

Where:

nmi	Nautical miles	
S.R.	Specific range	nmi/lb.

From drag aerodynamic theory, weight is found only in the induced part of the drag equation. Therefore, total drag must change since it is the sum of parasite and induced drag. As weight increases, the specific range and endurance decreases. The speed for optimum range and endurance also increases. The effects are depicted in figure 4.20.

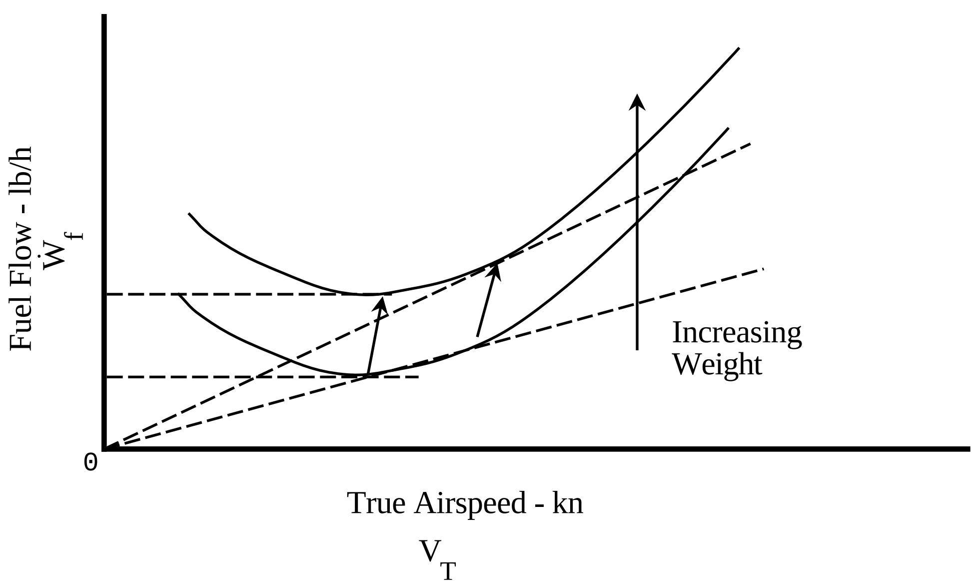

Figure 4.20
JET RANGE AND ENDURANCE WEIGHT EFFECTS

4.3.4.2 AERODYNAMIC EFFECTS

The external configuration determines the amount of parasite drag contribution to total drag. In this case, only the D_p of the drag curve is affected. Figure 4.21 depicts the parasite drag effects. For higher drag, the specific range and endurance time decrease. The speed for optimum range and endurance also decreases.

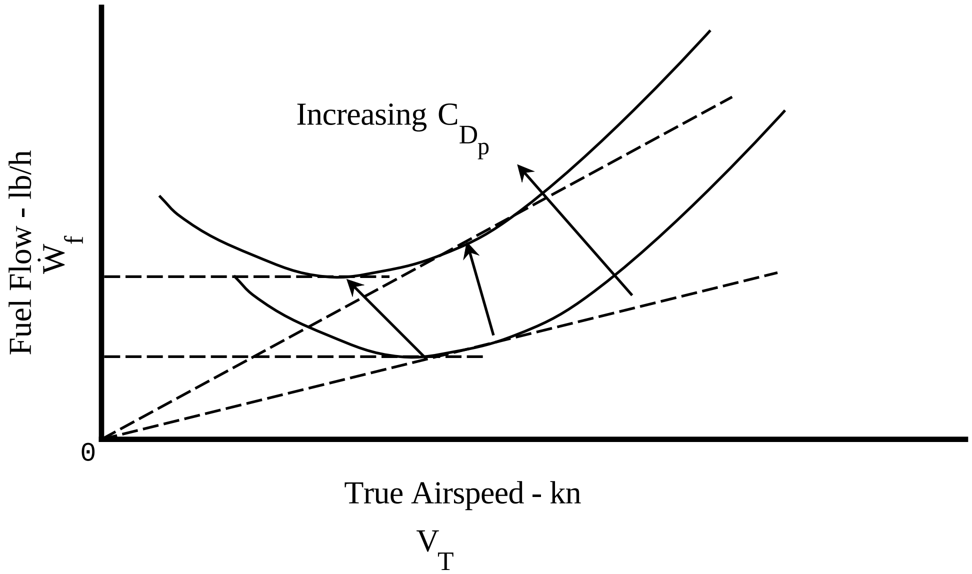

Figure 4.21
JET RANGE AND ENDURANCE PARASITE DRAG EFFECTS

4.3.4.3 ALTITUDE EFFECTS

The low speed drag equation does not contain altitude dependent variables; therefore, changes in altitude do not affect the drag curve. Minimum drag occurs at higher airspeed as altitude is increased. So long as TSFC does not increase markedly, a continuous gain of range is experienced as altitude is gained. Actually, up to the stratosphere, TSFC tends to decrease for most engines so greater gains in range are obtained than would be found if TSFC is assumed constant. At low altitudes, inefficient part throttle operation further increases the obtainable TSFC, producing an additional decrease in range. At high altitudes (above 35,000 to 40,000 feet) TSFC starts to increase so test data reveal a leveling off in range for stratospheric conditions. As airplanes fly higher, this leveling off results in an optimum best range altitude for any given gross weight, above which altitude, decreases in range are encountered. Figure 4.22 shows specific range has sizeable increases with altitude while endurance performance remains constant. A limiting factor on increased altitude would be transonic drag.

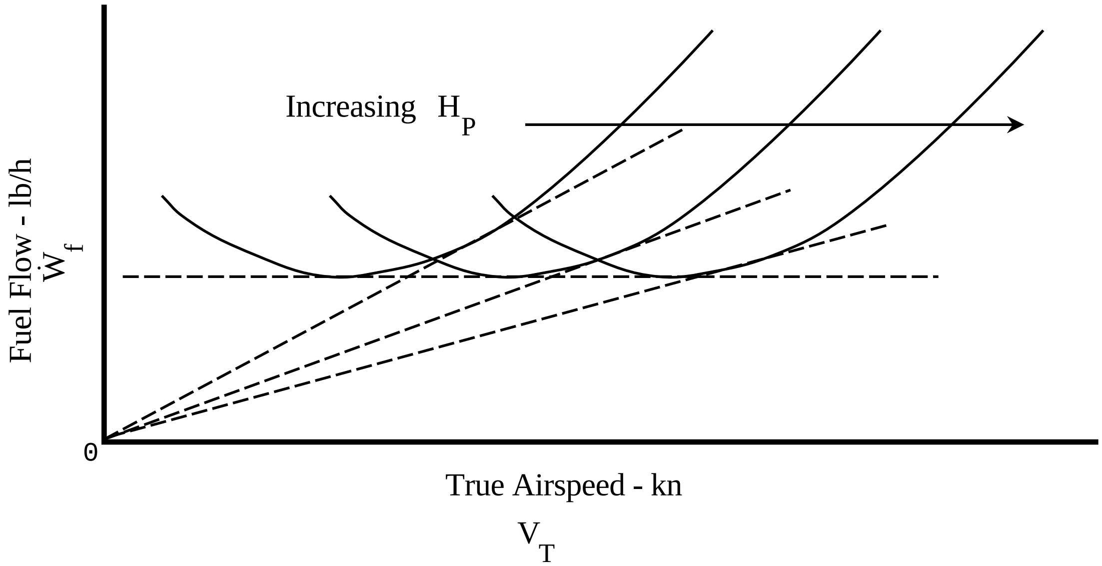

Figure 4.22
JET RANGE AND ENDURANCE ALTITUDE EFFECTS

4.3.5 TURBOPROP THRUST REQUIRED

Power requirements for turboprops deal with thrust horsepower rather than engine thrust as in the turbojet. The thrust horsepower required depends upon drag (thrust) and true airspeed. Thrust horsepower can be defined as:

$$THP = \frac{T\,V_T}{550} = \frac{D\,V_T}{550} \qquad \text{(Eq 4.57)}$$

The drag equation was given in Eq 4.23. Substituting for drag in Eq 4.57:

$$THP = \frac{C_{D_P}\,\rho_a\,V_T^3\,S}{1100} + \frac{W^2}{275\,\pi\,e\,AR\,S\,\rho_a\,V_T} \qquad \text{(Eq 4.58)}$$

Assumptions for Eq 4.58 are the same as for the drag equation:

1. Coefficient of drag is a function of lift and not a function of Mach number or Reynold's number.
2. A parabolic polar as in Eq 4.11.

3. Level flight with no thrust lift (L = W).

Eq 4.58 can be analyzed by making the following assumptions:

1. Fixed configuration and gross weight.
2. Constant altitude.
3. Low Mach.
4. S, C_{D_p}, and $\frac{1}{\pi eAR}$ are constants.

Thrust horsepower required can be expressed as:

$$THP = THP_p + THP_i \tag{Eq 4.59}$$

and graphically depicted as in figure 4.23.

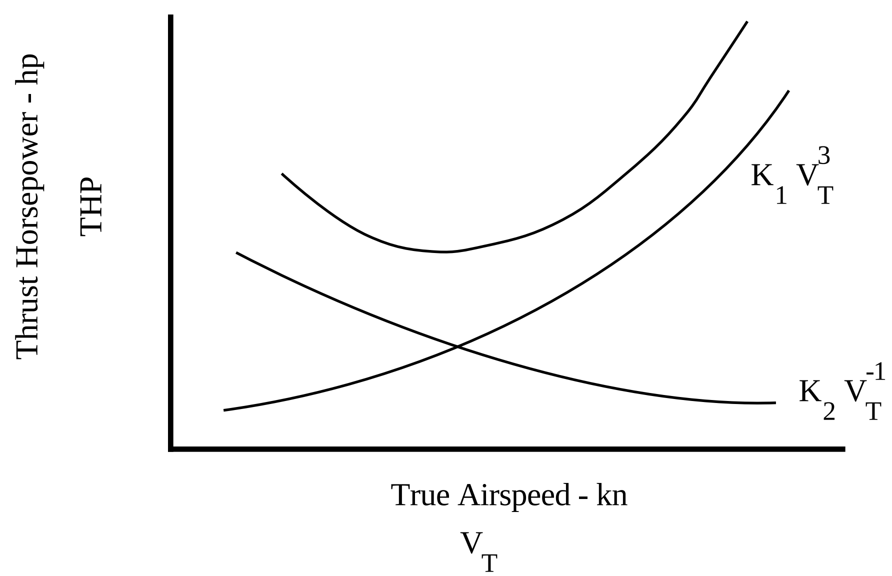

Figure 4.23
THRUST HORSEPOWER REQUIRED

A general form of the power required equation, not analytically based on a drag polar, can be developed. It starts with level flight horsepower required as given in Eq 4.57. In level flight, drag can be expressed as:

$$D = \frac{C_D}{C_L} W \qquad \text{(Eq 4.60)}$$

Substituting Eq 4.60 into Eq 4.57 for D, and assuming no vertical thrust:

$$THP = \frac{C_D\, W\, V_T}{C_L\, 550} \qquad \text{(Eq 4.61)}$$

$$L = W = C_L \frac{1}{2} \rho_a V_T^2 S \qquad \text{(Eq 4.62)}$$

$$THP = \frac{\left(\frac{2}{S}\right)^{\frac{1}{2}} W^{\frac{3}{2}} C_D}{\rho_a^{\frac{1}{2}} C_L^{\frac{3}{2}}\ 550} \qquad \text{(Eq 4.63)}$$

A general form of the level flight power required can be obtained by converting ambient density and substituting into Eq 4.63:

$$\rho_a = \rho_{ssl}\, \sigma \qquad \text{(Eq 4.64)}$$

$$THP = \frac{\sqrt{2}\, W^{\frac{3}{2}} C_D}{\left(S\, \rho_{ssl}\right)^{\frac{1}{2}} \sqrt{\sigma}\ C_L^{\frac{3}{2}}\ 550} \qquad \text{(Eq 4.65)}$$

The resultant equation shows THP as a function of C_L, W, σ. The effects of density (H_P and T_a) can be removed by defining a new term called equivalent thrust horsepower (THP_e) and substituting into Eq 4.65. THP_e is a function of C_L and W.

$$THP_e = THP \sqrt{\sigma} \qquad \text{(Eq 4.66)}$$

$$THP_e = \frac{\sqrt{2}\, W^{\frac{3}{2}}\, C_D}{\left(S\, \rho_{ssl}\right)^{\frac{1}{2}} C_L^{\frac{3}{2}}\ 550}$$

(Eq 4.67)

Where:

550	Conversion factor	$550\ \frac{\text{ft-lb}}{\text{s}} = 1$ horsepower
AR	Aspect ratio	
C_D	Drag coefficient	
C_{D_p}	Parasite drag coefficient	
C_L	Lift coefficient	
D	Drag	lb
e	Oswald's efficiency factor	
L	Lift	lb
π	Constant	
ρ_a	Ambient air density	slugs/ft^3
ρ_{ssl}	Standard sea level air density	0.0023769 slugs/ft^3
S	Wing area	ft^2
σ	Density ratio	
T	Thrust	lb
THP	Thrust horsepower	hp
THP_e	Equivalent thrust horsepower	hp
THP_i	Induced thrust horsepower	hp
THP_p	Parasite thrust horsepower	hp
V_T	True airspeed	kn
W	Weight	lb.

4.3.5.1 MINIMUM THRUST HORSEPOWER REQUIRED

Two methods exist to determine conditions for minimum power required in level flight. One is used for an airplane with a parabolic polar as in Eq 4.58. Substituting constants for the parasite and induced drag constants in Eq 4.58 yields:

$$THP = K_1 V_T^3 + K_2 V_T^{-1} \qquad \text{(Eq 4.68)}$$

Taking the derivative and setting it to zero for minimum power gives:

$$3 K_1 V_T^2 - K_2 V_T^{-2} = 0 \qquad \text{(Eq 4.69)}$$

Minimum power required can be found by multiplying Eq 4.69 by V_T and rearranging:

$$3 K_1 V_T^3 = K_2 V_T^{-1} \qquad \text{(Eq 4.70)}$$

$$3\, THP_p = THP_i \qquad \text{(Eq 4.71)}$$

Multiplying by $\frac{550}{V_T}$ and dividing both sides by qS:

$$3\, D_p = D_i \qquad \text{(Eq 4.72)}$$

$$3\, C_{D_p} = C_{D_i} \qquad \text{(Eq 4.73)}$$

Eq 4.73 relates directly to the parabolic drag polar and identifies a unique point on the drag polar producing the minimum power required in level flight. In figure 4.24 there is an optimal C_L to fly for minimum power required similar to the optimal C_L to fly for minimum drag. Also the minimum power point occurs at higher C_L than the minimum drag point.

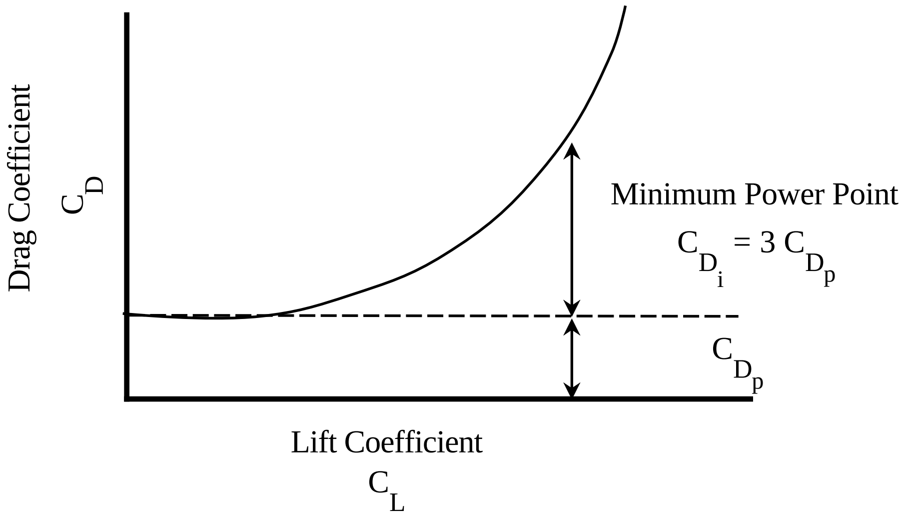

Figure 4.24
PARABOLIC DRAG POLAR

The second method to determine minimum power required involves using general conditions. By using the general form of the power required equation, Eq 4.65 reduces to a constant, K_o, and a relationship between lift and drag coefficients for a given weight and density altitude:

$$THP = K_o \frac{C_D}{C_L^{\frac{3}{2}}} \qquad \text{(Eq 4.74)}$$

If the point on the power polar is located which gives the maximum ratio of $\left.\frac{C_L^{3/2}}{C_D}\right|_{max}$, the power required is minimum as shown in figure 4.25.

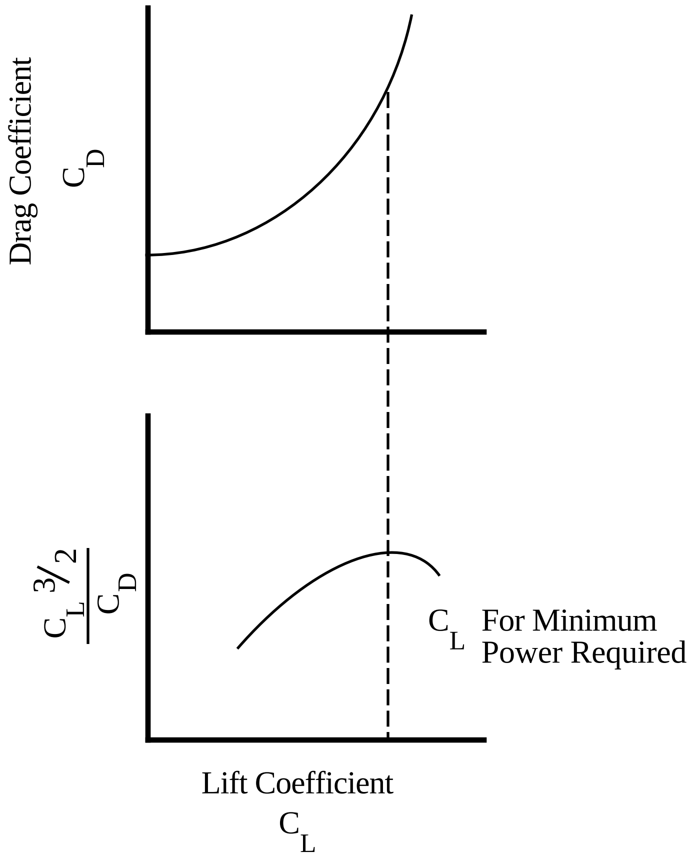

Figure 4.25
GENERAL POLAR

Power required data can be gathered without regard to altitude and generalized as THP_e versus V_e. There is also a method to determine the precise shape of the power required curve using only a small number of data points. Since most turboprops have thick wings, high AR, and little wing sweep; a parabolic drag polar is assumed. Further, if the following relationships are used, multiplying both sides of Eq 4.58 by V_e, and substituting, Eq 4.58 can be rewritten as Eq 4.78:

$$V_T = \frac{V_e}{\sqrt{\sigma}} \qquad \text{(Eq 4.75)}$$

$$THP = \frac{THP_e}{\sqrt{\sigma}} \qquad \text{(Eq 4.76)}$$

$$\sigma = \frac{\rho_a}{\rho_{ssl}} \qquad \text{(Eq 4.77)}$$

$$THP_e\, V_e = \frac{C_{D_P}\, \rho_{ssl}\, V_e^4\, S}{1100} + \frac{W^2}{275\, \pi\, e\, AR\, S\, \rho_{ssl}} \qquad \text{(Eq 4.78)}$$

Where:

AR	Aspect ratio	
C_D	Drag coefficient	
C_{D_i}	Induced drag coefficient	
C_{D_p}	Parasite drag coefficient	
C_L	Lift coefficient	
D_i	Induced drag	lb
D_p	Parasite drag	lb
e	Oswald’s efficiency factor	
K_1	Parasite drag constant	
K_2	Induced drag constant	
K_o	Constant	
π	Constant	
ρ_a	Ambient air density	$slugs/ft^3$
ρ_{ssl}	Standard sea level density	$slugs/ft^3$
S	Wing area	ft^2
σ	Density ratio	
THP	Thrust horsepower	hp
THP_e	Equivalent thrust horsepower	hp
THP_i	Induced thrust horsepower	hp
THP_{min}	Minimum thrust horsepower	hp
THP_p	Parasite thrust horsepower	hp
V_e	Equivalent airspeed	kn
V_T	True airspeed	kn
W	Weight	lb.

Figure 4.26 shows flight data for one gross weight plotted in the form of Eq 4.78. Data provided in this manner allows for easy interpretation. Some correction for varying aircraft weight may be required.

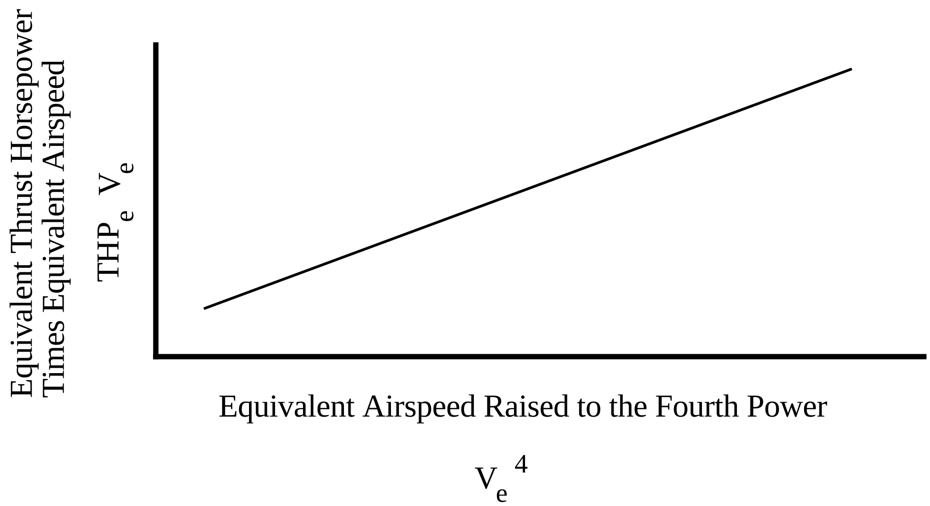

Figure 4.26
LINEARIZED THRUST HORSEPOWER REQUIRED

4.3.6 TURBOPROP RANGE AND ENDURANCE

For both a turboprop and reciprocating engine aircraft, the fuel flow is approximately proportional to the horsepower produced. This power is noted as either brake horsepower (BHP) or shaft horsepower (SHP). Additionally, shaft horsepower is related to thrust horsepower by propeller efficiency (η_P).

$$\eta_P = \frac{THP}{SHP} \tag{Eq 4.79}$$

$$THP = \eta_P \; SHP \tag{Eq 4.80}$$

Specific fuel consumption can be defined in either THP or SHP terms.

$$THPSFC = \frac{\dot{W}_f}{THP} \tag{Eq 4.81}$$

$$\text{SHPSFC} = \frac{\dot{W}_f}{\text{SHP}} \qquad \text{(Eq 4.82)}$$

$$\dot{W}_f = \frac{\text{THP}}{\eta_P}\ \text{SHPSFC} \qquad \text{(Eq 4.83)}$$

Where:

η_P	Propeller efficiency	
SHP	Shaft horsepower	hp
SHPSFC	Shaft horsepower specific fuel consumption	$\frac{\text{lb/h}}{\text{hp}}$
THP	Thrust horsepower	hp
THPSFC	Thrust horsepower specific fuel consumption	$\frac{\text{lb/h}}{\text{hp}}$
$\dot{W}_f$	Fuel flow	lb/h.

The relationship is similar to the TSFC versus net thrust for a jet in figure 4.19 and is shown in figure 4.27.

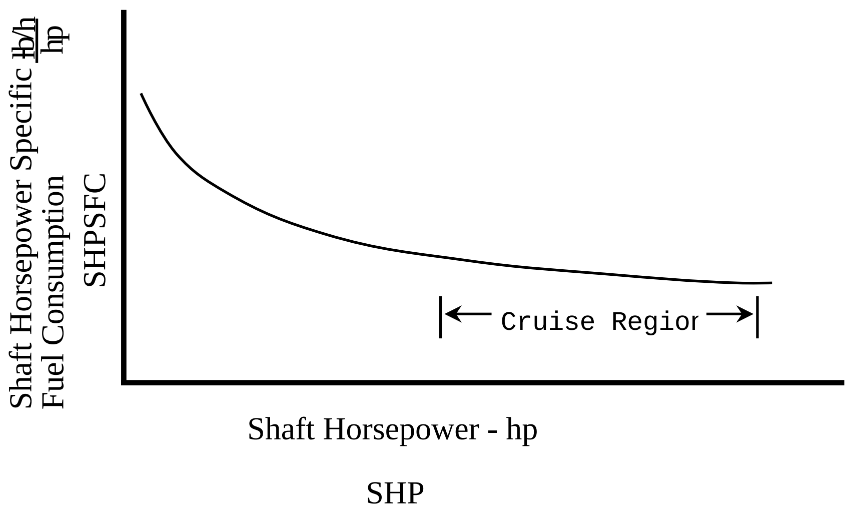

Figure 4.27
SHAFT HORSEPOWER SPECIFIC FUEL CONSUMPTION

Propeller efficiency varies with propeller advance ratio and pitch angle as shown in figure 4.28.

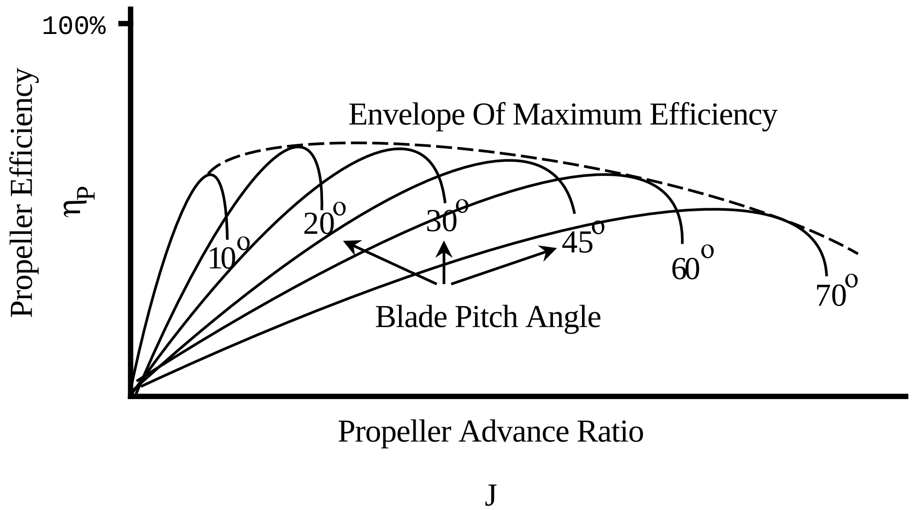

Figure 4.28
PROPELLER EFFICIENCY

If SHPSFC is fairly constant in the cruise region as in figure 4.27, and if η_P is held constant by changing blade pitch as in figure 4.28, then fuel flow is proportional to thrust horsepower and figure 4.29 can be developed.

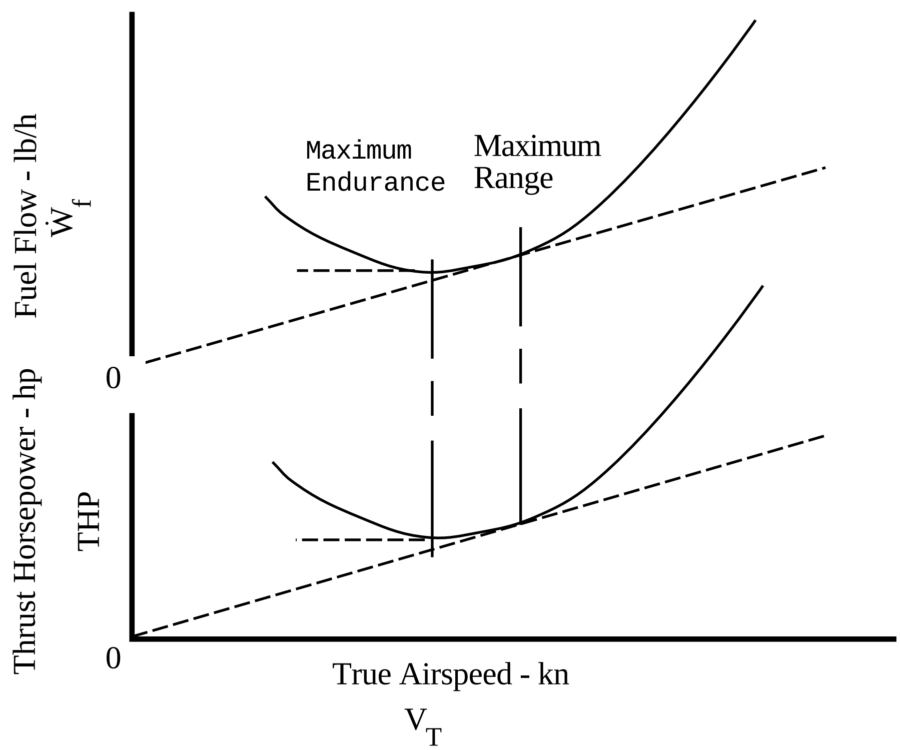

Figure 4.29
TURBOPROP RANGE AND ENDURANCE

Maximum range occurs at the tangent to the power required curve and maximum endurance occurs at the minimum power required point. This is very similar to the jet in figure 4.26.

4.3.6.1 WEIGHT AND AERODYNAMIC EFFECTS

As in the jet, power required changes with changes in aircraft weight or configuration as discussed in paragraphs 4.3.4.1 and 4.3.4.2.

4.3.6.2 ALTITUDE EFFECTS

In Eq 4.58, the density term (ρ_a) appears in both the parasite and induced terms. An increase in density altitude decreases parasite power required and increases induced power required. As in the jet, turboprop efficiency increases with engine speed as altitude is increased to an optimum then begins to decrease. Therefore, specific range is not constant, but increases with increasing altitude to an optimum. Endurance is not constant and increases to an optimum altitude. The results appear in figure 4.30.

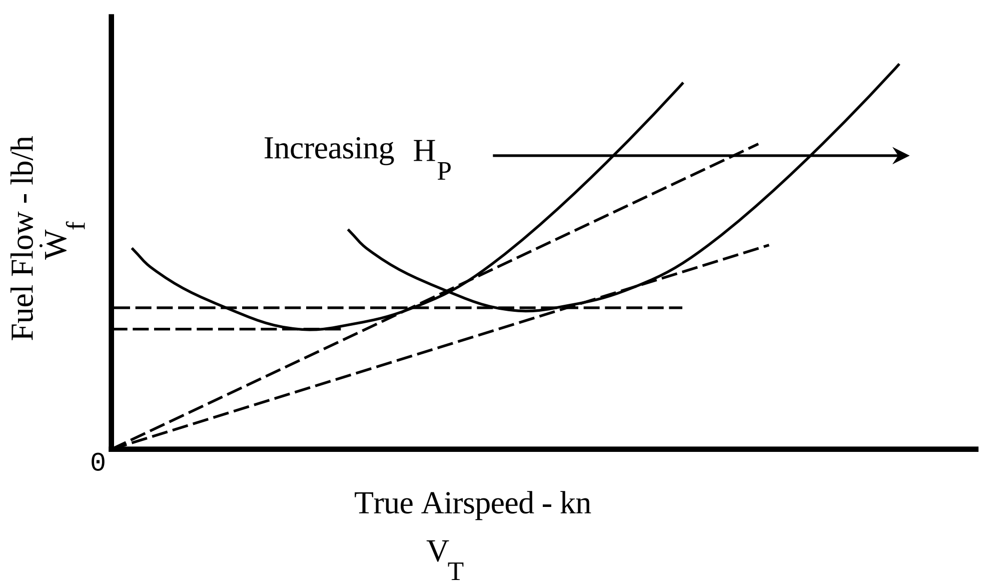

Figure 4.30
TURBOPROP RANGE AND ENDURANCE ALTITUDE EFFECTS

4.4 TEST METHODS AND TECHNIQUES

4.4.1 CONSTANT W/δ

The speed power flight test is a common method used to obtain the cruise performance of jet aircraft. This method allows determining maximum endurance, maximum range, and maximum airspeed in a minimum of flight sorties. The method involves gathering fuel flow data at various altitudes, gross weights, and airspeeds to define sufficiently the operating envelope of the aircraft. One W/δ is chosen for each set of data points, usually based on a nominal standard weight. The resulting curves do not represent all altitude and gross weight combinations which result in the particular W/δ of the curve. For example, an aircraft weighing 100,000 lb at an altitude of 18,000 ft has the same W/δ as a 200,000 lb aircraft at sea level. However, the fuel flow at 18,000 ft is much less than at sea level, resulting in a different fuel flow versus Mach curve. The Range Factor (specific range multiplied by the aircraft's weight) is the same in both of the cases above. Results appear as in figure 4.6.

The test techniques for the constant W/δ test are presented primarily for the single spool compressor, constant geometry engine. However, they apply equally well to twin spool compressor and variable geometry engines. The data reduction, on the other hand, applies only to fixed geometry engines. The power parameter used in this method is engine speed. The method is modified for more complex engines. Because of the variety of configurations existing, it is not practical, to describe methods for correcting engine data to standard conditions suitable for all types. The characteristics of each of the more complex engines may require different correction methods. Engine manufacturer's charts are a good source of data when making this analysis.

Target W/δ should cover the entire flight envelope. Data are gathered from M_{mrt} or V_{max} to V_{min} over a range of W/δ which covers the highest weight / highest altitude to lowest weight / lowest altitude. A rule of thumb for the altitude increments is to use 5,000 ft altitude intervals at standard gross weight. The actual number of flights is frequently limited by flight time or funding constraints. The interval may therefore need to be increased if flight tests are limited.

To gather data at a constant W/δ, a card is used to present target altitude as a function of fuel remaining. Values on the card would reflect instrument and position errors.

For example; W/δ is based on H_{P_c}, so altimeter position error and altimeter instrument correction must be subtracted to obtain the target H_{P_o} used when obtaining fuel flow data. The following data are required in preparing the W/δ card:

1. The empty weight of the aircraft.
2. Fuel density and fuel loading.
3. Altimeter calibrations relevant to altitude and airspeed of the test.
4. Airspeed calibrations (position and instrument errors).
5. Determine the target W/δ based upon standard gross weight and altitude range of interest.
6. Construct a plot of H_{P_c} versus weight as in figure 4.31. Given a weight, this plot can be used to determine altitude to fly to achieve the desired W/δ.
7. Convert gross weight into fuel remaining or used during the mission. Plot H_{P_c} versus fuel used as shown in figure 4.32. The dashed lines shown are the $\pm$2% W/δ variation permitted. If fuel temperature changes throughout the flight, use an average value for determining fuel density.
8. Since the values read from the figure are H_{P_c}, apply altimeter position error and instrument corrections to H_{P_c} to obtain H_{P_o} values for the data card. Pay particular attention to the sign of the correction because the above procedure necessitates going from calibrated values to observed values.
9. Prepare a data card similar to figure 4.33.

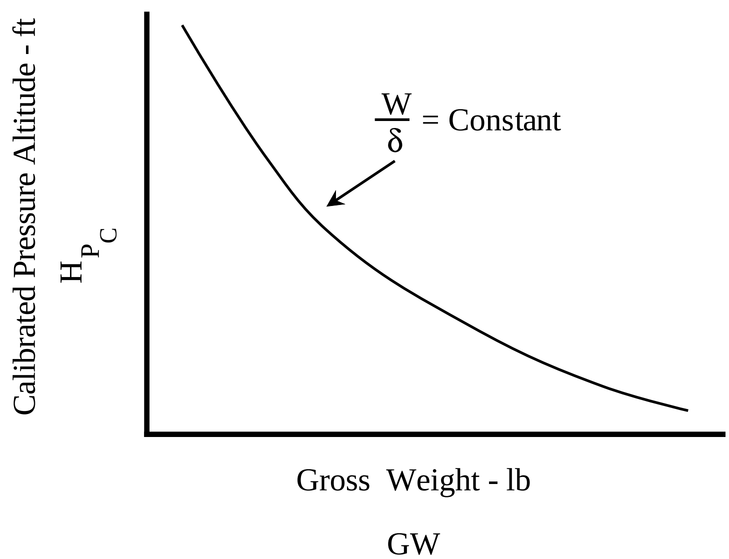

Figure 4.31
TEST ALTITUDE VERSUS GROSS WEIGHT

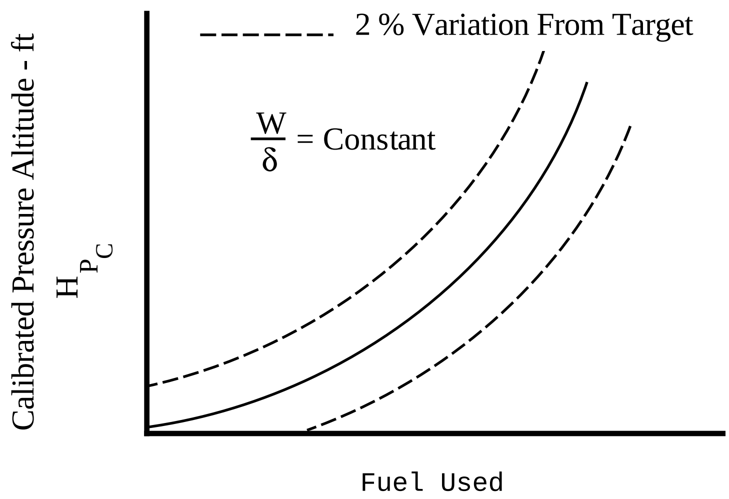

Figure 4.32
TEST ALTITUDE VERSUS FUEL USED

$V_{o\ tgt}$	$V_{o\ act}$	H_{P_c}	W_f start	W_f end	t	T_a	N	$\dot{W}_f$
kn	kn	ft	lb	lb	s	˚C	%	lb/h
V_{max}								

Figure 4.33
JET IN FLIGHT DATA CARD

As fuel weight changes, altitude changes to keep the target W/δ constant. The following equations show δ is directly related to fuel weight and is a linear function.

$$\left(\frac{W}{\delta}\right)_{Target} = \frac{W + W_f}{\delta} \qquad \text{(Eq 4.84)}$$

$$\left(\frac{W}{\delta}\right)_{Target} = \frac{W_{aircraft} + W_f}{\delta} \qquad \text{(Eq 4.85)}$$

$$\delta = \frac{W_{aircraft}}{\left(\frac{W}{\delta}\right)_{Target}} + \frac{1}{\left(\frac{W}{\delta}\right)_{Target}} W_f \qquad \text{(Eq 4.86)}$$

Ambient temperature (T_a) at altitude can be obtained in either of two ways:

1. Observed T_a from a balloon sounding, buddy system, etc.
2. Calculate T_a from measured OAT using the equation:

$$OAT = T_a \left(1 + \frac{\gamma - 1}{2} K_T M^2\right) \qquad \text{(Eq 4.87)}$$

Where:

δ	Pressure ratio	
γ	Ratio of specific heats	
K_T	Temperature recovery factor	
M	Mach number	
OAT	Outside air temperature	˚C or ˚K
T_a	Ambient temperature	˚C or ˚K
W	Weight	lb
$W_{aircraft}$	Aircraft weight	lb
W_f	Fuel weight	lb.

There are no special calibrations to the aircraft fuel quantity system although special instrumentation measuring fuel used can be calibrated. The zero fuel gross weight is contained in the aircraft weight and balance forms. In any event, the known quantity of fuel at engine start is necessary and close tracking of fuel remaining is required. For aircraft fuel tanks without quantity measurement, further special planning is required. One technique is to keep these fuel tanks full until their entire contents can be transferred to internal tanks with fuel quantity readings.

The derivation of Eq 4.37 assumed constant aircraft and engine geometry and the effects of changing Reynold’s number were small. The effect of decreased Reynold’s number may in fact not be small. The test sequence must be planned so as not to hide or mask the effect. The Reynold’s number effects can be checked by going back later in the flight and repeating data points at the same W/δ and Mach, but now at a lighter weight and therefore higher altitude. Figure 4.34 shows the results.

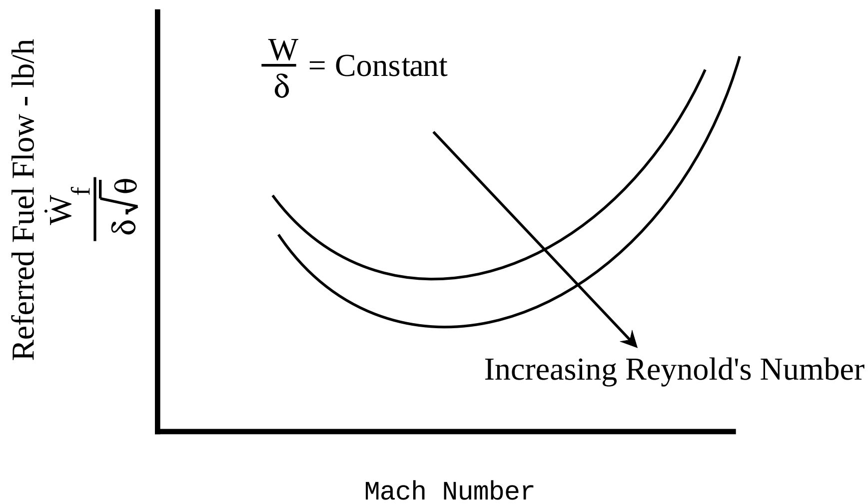

Figure 4.34
REYNOLD’S NUMBER EFFECT

The following is an efficient way for obtaining data at constant W/δ.

1. Technique:

a. The first point is flown at M_{mrt} or V_{max} with subsequent data taken in order of descending Mach or airspeed. Stabilization is quicker if the data point is approached from the fast side. If a point is required faster than the present speed, accelerate beyond the target speed and then decelerate again into the target point.

b. Ideally, the fuel remaining for the correct W/δ occurs midway through the timed period. For low fuel flow rates a longer stable point may be required.

c. The constant altitude technique can be used on the front side of the power curve, and the constant airspeed technique used on the back side. On the front side, fix the engine power, adjust attitude, then maintain altitude to obtain stabilized airspeed. On the back side, stabilize on airspeed by adjusting attitude then adjust power to maintain constant altitude. For the airspeeds near the airspeed for minimum power, a combination of the two techniques may be used although the back side technique is more useful if airspeed is the more difficult parameter to hold constant.

d. Stabilize the aircraft by proper trimming and control pitch by outside reference. Record data in an organized sequence to expedite the data point. Trim the aircraft for hands off flight when stabilized.

2. Procedure:

a. Before engine start, know the correct fuel loading.

b. When approaching within 2000 to 3000 feet of the test altitude, read the fuel remaining and extrapolate to account for the time required to stabilize on the data point. Determine the target altitude for the first data point.

c. Using the target airspeed from the flight data card, apply the altimeter position error and instrument correction to determine the H_{P_o}. Using the allowed fuel, establish a stable point at the target airspeed and correct altitude.

d. Obtain enough stabilized points to completely define the fuel flow versus velocity curve at the particular W/δ tested.

e. Record the fuel counter reading and start the stop watch when the speed is stabilized and the aircraft is within 2% of the desired W/δ. Fly the aircraft at the required altitude for a minimum of one minute, record the fuel counter reading and other data. If the airspeed changes more than 2 knots using the front side technique or the altitude change exceeds 50 ft using the back side technique, repeat the point.

f. Data are recorded in wings level, ball-centered, level (no vertical velocity), stabilized (unaccelerated) flight. Data are recorded starting with the most important first; fuel flow, pressure altitude, airspeed, ambient temperature, fuel weight/gross weight. Angle of attack is nice to have and, engine speed, EGT, EPR, etc., can provide correlating information. A minimum of six points are flown at each W/δ. Fly extra points where the minimum range and endurance area are expected from pre-flight planning.

4.4.1.1 W/δ FLIGHT PLANNING PROGRAM

A constant W/δ flight planning program generates a list of altitudes required to attain a desired W/δ in flight based on fuel remaining.

The program lists numerous performance routines including level flight. From the menu, select the appropriate name of the W/δ flight planning program. Instructions to proceed would be a series of questions to be answered. The following information is required as inputs:

1. Aircraft zero fuel weight (includes everything except fuel, pilots, stores, etc.).
2. Minimum and maximum fuel weight for calculations.
3. Fuel weight increment for calculations.
4. Desired referred weight (W/δ).

The program calculation includes:

1. Gross weight for each increment of fuel by adding the zero fuel weight to the fuel remaining.
2. Pressure ratio for each gross weight above by dividing the given gross weight by the target W/δ desired.
3. Selection of H_{P_c} for each δ as would be done manually from an atmospheric table.

Data can be output as a listing of the calibrated pressure altitude required to maintain the constant W/δ for each increment of fuel remaining. Instrument error and position error are unique to the installation and instrument; therefore, corrections must be made to the H_{P_c} in flight to determine the H_{P_o} required for the given W/δ. The program calculates the pressure altitude required to attain the target W/δ for each increment of fuel remaining. It should present the information on a monitor or in some form which can be printed. After review of the prepared information any desired changes would be made and the W/δ planning record would be printed for use in flight. An example W/δ planning record is presented in figure 4.35.

Aircraft:			W/δ:	42285	
Pilot:					
W_f	H_P	W_f	H_P	W_f	H_P
3200	31175				
3100	31359				
3000	31543				
2900	31729				
2800	31916				
2700	32105				
2600	32295				
2500	32486				
2400	32679				
2300	32873				
2200	33068				
2100	33265				
2000	33463				
Instrument Correction				Position Error	
H_P	$\Delta H_{P_{ic}}$			V_o, M_o	ΔH_{pos}

Figure 4.35
W/δ PLANNING CHART

4.4.1.2 DATA REQUIRED

H_P, V_o, T_a, and W_f or GW are necessary to complete the test results. Angle of attack is desired information and, engine speed, EGT, EPR, etc., can provide correlating information. A minimum of six points are flown at each W/δ with most of the points taken on the front side and bucket.

4.4.1.3 TEST CRITERIA

1. Balanced (ball centered), wings level, unaccelerated flight.
2. Engine bleed air systems off or operated in normal flight mode.
3. Stabilized engine power setting.
4. Altimeter set at 29.92.

4.4.1.4 DATA REQUIREMENTS

1. Stabilize 60 seconds prior to recording data.
2. Record stabilized data for 30 seconds.
3. W/δ within ± 2% of the target.
4. Airspeed change less than 2 kn in one minute for front side.
5. Altitude change less than 50 ft in one minute for back side.

4.4.2 RANGE CRUISE TEST

The range cruise test (ferry mission) is used to verify and refine the estimates of the range performance generated during the W/δ tests. Specifically, use it to check the optimum W/δ and the total ferry range. Usually a series of flights are flown at W/δ above and below the predicted optimum W/δ. The standard day range from each of these can be used to determine the actual optimum W/δ and are compared with the data predicted by W/δ testing.

Planning the time or distance flown accounts for all portions of the test including cruise as show in figure 4.36.

	Fuel used	Fuel remaining
Prior to engine start		
Engine start and taxi		
Takeoff and accelerate to climb schedule		
Climb		
Cruise		
Fuel reserve		

Figure 4.36
FUEL USED FOR CRUISE

Estimated fuel used for engine start, taxi, takeoff, and acceleration to climb schedule is obtained from manufacturer's charts. Fuel used in the climb is obtained from the prior climb tests. Fuel reserve is determined by Naval Air System Command Specification, AS-5263, reference 2. The total of these fuel increments subtracted from the total fuel available gives the amount of fuel available for the cruise portion of the test.

Plan fuel used during the climb corresponding to W/δ altitude. When on cruise schedule, record data often enough to obtain at least 10 points.

Figure 4.37 is a sample flight data card for use on the test flight.

Data Point	Time	F/C	V_o	H_{P_o}	T_a	N	$\dot{W}_f$
Prior eng start							
Start							
Taxi							
Takeoff							
Start climb							
End climb							
Start cruise							
Cruise increment							
Cruise increment							
End cruise							

Figure 4.37
CRUISE DATA CARD

A recommended procedure for performing the range cruise test is:

1. Prior to engine start, check that the correct amount of fuel is onboard and the fuel counter is set correctly.

2. Record data at each point planned, i.e., engine start, taxi etc.

3. Upon reaching the altitude corresponding to the fuel counter reading for the desired W/δ, set up the cruise climb at the desired Mach.

4. Increase altitude as the fuel counter is decreased to maintain a constant W/δ by performing a shallow climb. An alternate method is to hold a constant altitude and stair-step the aircraft in increments of 100 to 200 feet. Cruise begins in the stratosphere and the Mach and engine speed remain constant (using a constant velocity corresponding to the indicated Mach is preferred due to the accuracy of the instruments). If cruise begins below the tropopause, a slight decrease in engine speed is required initially. $\frac{N}{\sqrt{\theta}}$ is a function of W/δ and Mach. For a given W/δ and Mach, a constant $\frac{N}{\sqrt{\theta}}$ is required and therefore engine

speed is decreased as T_a decreases to hold $\frac{N}{\sqrt{\theta}}$ constant. During the cruise portion, minimize throttle movements. If turns are required, make them very shallow (less than 10˚ bank).

4.4.2.1 DATA REQUIRED

Obtain a sufficient number of data points enroute to minimize the effect of errors in reading the data. The data card suggested in paragraph 4.4.2 can be expanded to include incremental points along the cruise line. The following parameters are recorded:

H_{P_o}, V_o, T_o, Aircraft weight, time.

4.4.2.2 TEST CRITERIA

1. Balanced (ball centered), wings level, unaccelerated flight.
2. Engine bleed air systems off or operated in normal flight mode.
3. Stabilized engine power setting.
4. Altimeter set at 29.92 in.Hg.

4.4.2.3 DATA REQUIREMENTS

1. Stabilize 60 seconds prior to recording data.

then:

2. Airspeed change less than 2 kn for front side.
3. Altitude change less than 50 ft for back side.
4. W/δ within 2% of target.

4.4.3 TURBOPROP RANGE AND ENDURANCE

The turboprop level flight tests do not require the rigors associated with the W/δ method most often used for the turbojet (although the W/δ method can be used). Measurement of level flight parameters only require setting power for level flight and recording data. Power settings should be those recommended by the engine manufacturer for the altitude selected. An iterative computer program would be used to develop referred curves from which range and endurance for any altitude can be determined. Test day data would use power required plotted against airspeed to determine the range and endurance airspeeds for a given altitude. Test day data would only be useful for flights at the exact conditions from which the data was obtained and would not be of much use in determining the level flight performance under any other condition.

Shaft horsepower needs to be determined for each level flight point flown. Shaft horsepower is related to Thrust horsepower through propeller efficiency as stated in Eq 4.79.

$$\eta_P = \frac{THP}{SHP} \tag{Eq 4.79}$$

Neither THP nor SHP are normally available from cockpit instruments but can be determined from engine curves developed by the engine manufacturer. Torque, rpm, fuel flow, and ambient temperature would be used to determine the SHP. Fuel flow would be provided by the engine curves, measured in flight, or determined by fuel remaining and time aloft.

To gather data, a card is used to list the data attainable from standard cockpit instruments or special performance instrumentation if installed. Prepare a data card similar to figure 4.38.

$V_{i\ tgt}$ kn	$V_{o\ act}$ kn	H_{P_c} ft	$W_{f\ start}$ lb	$W_{f\ end}$ lb	t s	T_a ˚C	Q ft-lb	N RPM	$\dot{W}_f$ lb/h

Figure 4.38
PROP IN FLIGHT DATA CARD

Ambient temperature can be determined the same way as described in section 4.4.1 if not available in the cockpit.

There are no special calibrations to the aircraft fuel quantity system although special instrumentation measuring fuel used can be calibrated. The zero fuel gross weight is contained in the aircraft weight and balance forms. In any event, the known quantity of fuel at engine start is necessary and close tracking of fuel remaining is required. If fuel used during each point is small, just record the fuel remaining as data is being recorded instead of fuel at the beginning and end of the data point. For aircraft fuel tanks without quantity measurement, further special planning is required. One technique is to keep these fuel tanks full until their entire contents can be transferred to internal tanks with fuel quantity readings.

The exact number of data points to fly is not important although a sufficient number of points should be flown over the airspeed range from V_{max} to near stall to lend confidence to the ensuing computer iterations when smoothing referred curves.

The constant altitude technique can be used on the front side of the power curve, and the constant airspeed technique used on the back side. On the front side, fix the engine power then adjust attitude, maintaining altitude, to obtain stabilized airspeed. On the back side, stabilize on airspeed by adjusting attitude then adjust power to maintain constant altitude. For the airspeeds near the airspeed for minimum power, a combination of the two techniques may be used although the back side technique is more useful if airspeed is most difficult to hold constant.

Using the target airspeed from the flight data card, apply the altimeter position error and instrument correction to determine the H_P. The corrections could be applied later since the test is not the W/δ technique. Stabilize the aircraft by proper trimming, control pitch by outside reference, and record data in an organized sequence to expedite the data point. Trim the aircraft for hands off flight when stabilized.

Data are recorded in wings level, ball-centered, level (no vertical velocity), stabilized (unaccelerated) flight. Data are recorded starting with the most important first; torque, engine speed, fuel flow if available, pressure altitude, airspeed, ambient temperature if available, fuel remaining / gross weight. Angle of attack is nice to have and, N_1, EGT, etc., can provide correlating information. Fly extra points where the bucket or maximum endurance area is expected from pre-flight planning since power changes for a given airspeed range might be quite small here.

4.4.3.1 DATA REQUIRED

$\dot{W}_f$, or fuel remaining and time of flight, torque, engine speed, T_a, V_o, H_{P_o} are necessary to complete the test results. Angle of attack, N_1, EGT, etc., can provide correlating information. The data points should be flown over the range of airspeeds attainable with emphasis taken on the front side and bucket.

4.4.3.2 TEST CRITERIA

1. Balanced (ball centered), wings level, unaccelerated flight.
2. Engine bleed air system off or operated in normal flight mode.
3. Stabilized engine power setting.
4. Altimeter set at 29.92.

4.4.3.3 DATA REQUIREMENTS

1. Stabilize 60 s prior to recording data.
2. Record stabilized data for 30 s.
3. Airspeed change less than 2 kn in one minute for front side.
4. Altitude change less than 50 ft in one minute for back side.

4.5 DATA REDUCTION

4.5.1 JET RANGE AND ENDURANCE

The following equations are used for referred range and endurance:

$$H_{P_c} = H_{P_o} + \Delta H_{P_{ic}} + \Delta H_{pos} \quad \text{(Eq 4.88)}$$

$$T_a = T_o + \Delta T_{ic} \quad \text{(Eq 4.89)}$$

$$V_c = V_o + \Delta V_{ic} + \Delta V_{pos} \quad \text{(Eq 4.90)}$$

$$\sigma = \frac{\rho_a}{\rho_{ssl}} \quad \text{(Eq 4.77)}$$

$$\delta = \frac{P_a}{P_{ssl}} \qquad \text{(Eq 4.43)}$$

$$\theta = \frac{T_a}{T_{ssl}} \qquad \text{(Eq 4.38)}$$

$$V_T = \frac{V_c}{\sqrt{\sigma}} \qquad \text{(Eq 4.91)}$$

$$M = \frac{V_T}{a_{ssl}\sqrt{\theta}} \qquad \text{(Eq 4.92)}$$

$$\dot{W}_{f_{ref}} = \frac{\dot{W}_f}{\delta\sqrt{\theta}} \qquad \text{(Eq 4.93)}$$

$$W_{ref} = \frac{W}{\delta} \qquad \text{(Eq 4.94)}$$

Where:

a_{ssl}	Standard sea level speed of sound	661.483 kn
δ	Pressure ratio	
$\Delta H_{P_{ic}}$	Altimeter instrument correction	ft
ΔH_{pos}	Altimeter position error	ft
ΔT_{ic}	Temperature instrument correction	˚C
ΔV_{ic}	Airspeed instrument correction	kn
ΔV_{pos}	Airspeed position error	kn
H_{P_c}	Calibrated pressure altitude	ft
H_{P_o}	Observed pressure altitude	ft
M	Mach number	
P_a	Ambient pressure	psf
P_{ssl}	Standard sea level pressure	2116.217 psf
θ	Temperature ratio	
ρ_a	Ambient air density	slugs/ft^3

ρ_{ssl}	Standard sea level air density	0.0023769 slugs/ft^3
σ	Density ratio	
T_a	Ambient temperature	˚C
T_o	Observed temperature	˚C
T_{ssl}	Standard sea level temperature	˚C
V_c	Calibrated airspeed	kn
V_o	Observed airspeed	kn
V_T	True airspeed	kn
W	Weight	lb
$\dot{W}_f$	Fuel flow	lb/h
$\dot{W}_{f\ ref}$	Referred fuel flow	lb/h
W_{ref}	Referred aircraft weight	lb.

From the observed airspeed, altitude, and temperature data, compute $\dot{W}_{f\ ref}$ and M as follows:

Step	Parameter	Notation	Formula	Units	Remarks
1	Observed altitude	H_{P_o}		ft	
2	Altitude instrument correction	$\Delta H_{P_{ic}}$		ft	Lab calibration
3	Altitude position error	ΔH_{pos}		ft	Airspeed calibration
4	Calibrated altitude	H_{P_c}	Eq 4.88	ft	
5	Observed temperature	T_o		˚C	
6	Temp instrument correction	ΔT_{ic}		˚C	Lab calibration
7	Ambient temperature	T_a	Eq 4.89	˚C	
8	Observed airspeed	V_o		kn	
9	Airspeed instrument correction	ΔV_{ic}		kn	Lab calibration
10	Airspeed position error	ΔV_{pos}		kn	Airspeed calibration
11	Calibrated airspeed	V_c	Eq 4.90	kn	

12	Ambient pressure	P_a		psf	From Appendix VI, or calculated from 4
13	Ambient air density	ρ_a		slugs/ft^3	From Appendix VI or calculated (see Chapter 2)
14	Density ratio	σ	Eq 4.77		Or from Appendix VI
15	Pressure ratio	δ	Eq 4.43		
16	Temperature ratio	θ	Eq 4.38		
17	True airspeed	V_T	Eq 4.91	kn	
18	Mach number	M	Eq 4.92		Or from Mach indicator with instrument correction
19	Referred fuel flow	$\dot{W}_{f\ ref}$	Eq 4.93	lb/h	
20	Referred weight	W_{ref}	Eq 4.94	lb	

Plot referred fuel flow as a function of Mach for each W/δ flown as shown in figure 4.39.

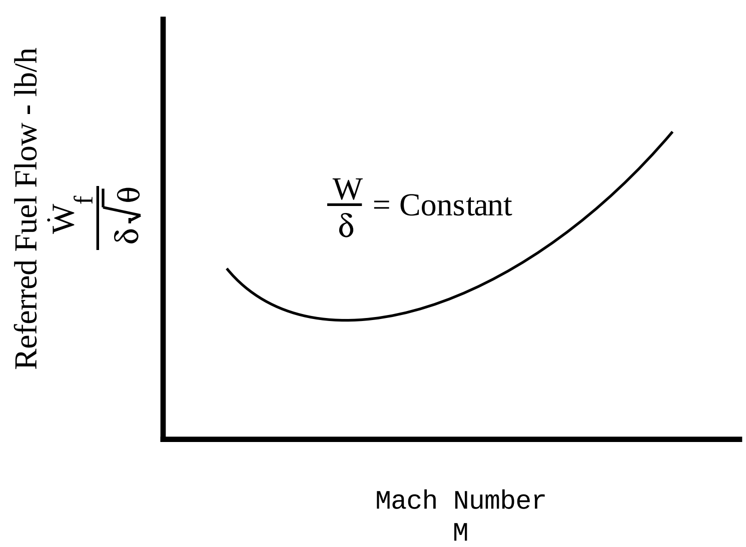

Figure 4.39
REFERRED LEVEL FLIGHT PERFORMANCE

4.5.2 JET FERRY RANGE

The following equations are used in determination of ferry range:

$$R_{Test} = \sum_{j=1}^{n} V_j \, \Delta t_j \qquad \text{(Eq 4.95)}$$

$$R.F._{Test} = \frac{t_T}{\ln\left(\frac{W_1}{W_2}\right)} \qquad \text{(Eq 4.96)}$$

$$R_{Std} = R.F._{Test} \ln\left(\frac{W_{Std_1}}{W_{Std_2}}\right) \qquad \text{(Eq 4.97)}$$

Where:

Δt_j	Time of each time interval	s
n	Number of time intervals	
$R.F._{Test}$	Test day average range factor	
R_{Std}	Standard day cruise range	nmi
R_{Test}	Test cruise range	nmi
t_T	Total cruise time	s
Vj	Avg true airspeed in time interval	kn
W_1	Initial cruise weight	lb
W_2	Final cruise weight	lb
W_{Std_1}	Standard initial cruise weight	lb
W_{Std_2}	Standard final cruise weight	lb.

Standard day cruise weight at the start and end of cruise is required to calculate ferry range. W_{Std_2} is the standard weight at the AS-5263 fuel reserve. Pitot static relationships are used to calculate true airspeed, V_T, and Mach. The test day W/δ for each set of data points is calculated to ensure the planned Mach and W/δ were flown. The test day total range (air miles) is found by numerically integrating the true airspeed with respect to time as in Eq 4.95. The test day average range factor is found from Eq 4.96. Standard day cruise range is then predicted using Eq 4.97.

The total range capability of the aircraft can be evaluated by adding the nautical air miles traveled during climb plus nautical air miles traveled during cruise. Range credit is not allowed for takeoff, acceleration to climb speed, and descent.

4.5.3 JET COMPUTER DATA REDUCTION

Various computer programs are in existence to assist in reduction of performance data. This section contains a brief summary of the assumptions and logic which might be used. The treatment is purposefully generic as programs change over time or new ones are acquired or developed. It is assumed detailed instructions on the use of the particular computer or program are available for the computer program to be used. In any event, the operating system would be invisible to the user.

4.5.3.1 BASIC DATA ENTRY

The purpose of the computer data reduction for range and endurance is to automatically calculate and generate referred range and endurance curves for data taken at one constant referred gross weight (W/δ). It would also combine curves for many W/δ flights into a composite curve. The program may be capable of predicting actual range and endurance performance at the specified W/δ for any temperature conditions.

From a menu selection the appropriate choice would be made to enter the W/δ Range and Endurance program. Data entry requirements for the program are cued as follows:

1. Basic Data:
    a. Type of aircraft (T-2C, P-3C).
    b. Bureau Number.
    c. Standard gross weight.
    d. Target altitude.
    e. Method (W/δ).
    f. Miscellaneous data (such as anti-ice on).
    g. Date of tests.
    h. Pilot(s) name(s).

2. For each data point:
    a. Point #.
    b. Observed airspeed (V_o).
    c. Observed pressure altitude (H_{P_o}).
    d. OAT or ambient temperature (T_a).
    e. Fuel flow.
    f. Gross weight.

Note the following:

1. Basic data is that common to all data points. Other data is entered for each point. Items 1.c and d are used for calculations. The rest of basic data is header information for the final plots.
2. V_o and H_{P_o} for each point are observed variables.
3. Temperature may be input as either ambient (degrees Kelvin) or as OAT (degrees Centigrade). This allows data from several different flights to be included in one file.
4. In multi-engine aircraft, fuel flow is the total of all engines.
5. Gross weight must be calculated from the fuel remaining plus the zero fuel weight of the aircraft. This allows data from more than one aircraft or flight to be included in the same file.
6. Edit and store data appropriately for future use or additions.
7. The process is repeated for each W/δ flown.
8. Data from a single W/δ curve are of limited usefulness until they are combined with data from other values of W/δ to cover the normal operating envelope of the aircraft. For example, a family of curves of referred fuel flow versus Mach for various values of constant W/δ may be used to calculate range and endurance for any flight conditions by interpolating between the curves.

One feature of the program calculates the variation between the actual W/δ and target W/δ as shown in figure 4.40. Any points falling outside of the allowed $\pm$ 2% band can be identified for future reference and editing or deletion.

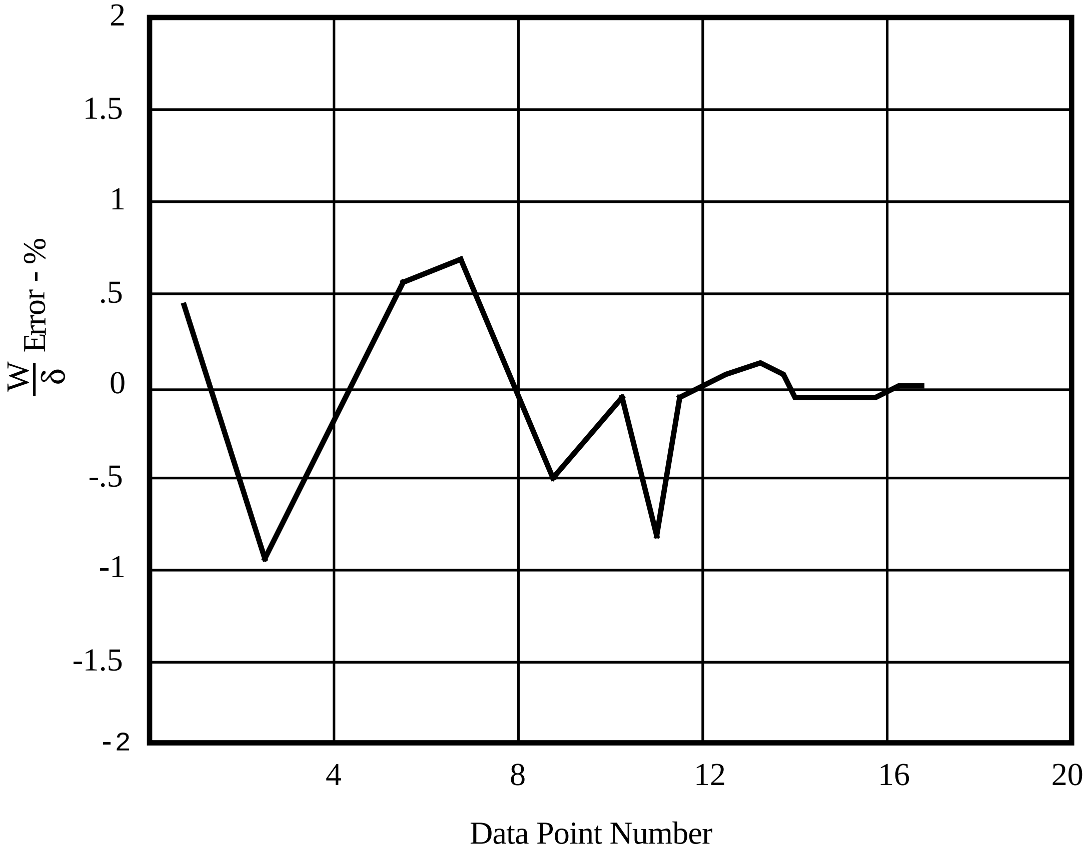

Figure 4.40
W/δ VARIATION

4.5.3.1.1 EQUATIONS USED

Position error calculations from calibrations:

$$H_{P_c} = H_{P_o} + \Delta H_{P_{ic}} + \Delta H_{pos} \qquad \text{(Eq 4.88)}$$

$$V_c = V_o + \Delta V_{ic} + \Delta V_{pos} \qquad \text{(Eq 4.90)}$$

True Mach:

$$M = f\left(V_c, H_{P_c}\right) \qquad \text{(Eq 4.98)}$$

Pressure ratio:

$$\delta = f\left(H_{P_c}\right) \qquad \text{(Eq 4.99)}$$

Test W/δ:

$$\frac{W + W_f}{\delta} = \frac{W}{\delta} \qquad \text{(Eq 4.100)}$$

% W/δ error from:

$$\frac{W}{\delta}\text{(error)} = \frac{100\left[\frac{W}{\delta}\text{(test)} - \frac{W}{\delta}\text{(target)}\right]}{\frac{W}{\delta}\text{(target)}} \qquad \text{(Eq 4.101)}$$

If ambient temperature (˚K) was entered:

$$^\circ C = {}^\circ K - 273.15 \qquad \text{(Eq 4.102)}$$

$$OAT = f\left(T_a, M\right) \qquad \text{(Eq 4.103)}$$

If OAT was entered:

$$T_a = f\left(OAT, M\right) \qquad \text{(Eq 4.104)}$$

Temperature ratio:

$$\theta = \frac{T_a}{T_{ssl}} \qquad \text{(Eq 4.38)}$$

Referred fuel flow:

$$\dot{W}_{f_{ref}} = \frac{\dot{W}_f}{\delta\sqrt{\theta}} \qquad \text{(Eq 4.93)}$$

Referred specific range:

$$S.R.\,\delta = \frac{661.483M}{\left(\frac{\dot{W}_f}{\delta\sqrt{\theta}}\right)} \qquad \text{(Eq 4.105)}$$

Where:

δ	Pressure ratio	
$\Delta H_{P_{ic}}$	Altimeter instrument correction	ft
ΔH_{pos}	Altimeter position error	ft
ΔV_{ic}	Airspeed instrument correction	kn
ΔV_{pos}	Airspeed position error	kn
H_{P_c}	Calibrated pressure altitude	ft
H_{P_o}	Observed pressure altitude	ft
M	Mach number	
OAT	Outside air temperature	˚C
θ	Temperature ratio	
S.R.	Specific range	nmi/lb
T_a	Ambient temperature	˚C or ˚K
T_{ssl}	Standard seal level temperature	˚C or ˚K
V_c	Calibrated airspeed	kn
V_o	Observed airspeed	kn
W	Weight	lb
W_f	Fuel weight	lb
$\dot{W}_f$	Fuel flow	lb/h
$\dot{W}_{f\ ref}$	Referred fuel flow	lb/h.

4.5.3.2 REFERRED CURVES

4.5.3.2.1 REFERRED FUEL FLOW VERSUS MACH

The program produces a plot of only the referred data points first. It then fairs the curve through the points based on the order of fit selected. In general, the lowest order fit reasonably representing the shape of the curve (3 rd order is a good starting point) is used. If any data points are clearly suspect, the program presents opportunities to re-enter the data subroutine for editing. This referred curve (similar to figure 4.39) can be combined with different W/δ curves to plot a family of referred fuel flow curves similar to figure 4.49. Single curves for each W/δ would include the actual data points plotted on them.

4.5.3.2.2 REFERRED SPECIFIC RANGE VERSUS MACH

The program plots referred specific range ((S.R.(δ)) versus Mach, calculated from the faired referred fuel flow curve as developed in paragraph 4.5.3.2.1. A single curve is plotted for each W/δ. The program plots a family of these curves as shown in figure 4.50 at various W/δs. The optimum range factor and maximum range of the aircraft would be calculated. Since these curves are derived from the faired curves no data points are shown.

4.5.3.2.3 MAXIMUM VALUES

A list of the referred maximum range and endurance points from each W/δ curve in the previous graphs can be displayed. These values are valid only for the target W/δs. They are useful in precisely locating the points on the referred plots. The maximum referred specific range points is used to calculate range factor in another option of the program. Data for each W/δ may look like:

1.	W/δ	21,000 lb.
2.	Maximum endurance:	Referred fuel flow = 1400 lb/h
		Mach = 0.32
3.	Maximum range:	Referred specific range = 0.02 nmi/lb
		Mach = 0.52

4.5.3.2.4 REFERRED PARAMETERS

A list of referred parameters calculated for each data point for each W/δ curve is available in the program routine. A check of the program's accuracy can be accomplished by hand-calculating one or two points. The parameters displayed are each individual data point, not points on the faired curves. The display may appear as in figure 4.41.

Nominal	Altitude	30000 ft					
Point #	V_c kn	H_{P_c} ft	T_a $^\circ$C	Mach true	$\dot{W}_{f\ ref}$ lb/h	W/δ lb	S.R.(δ) nmi/lb
1	285.5	27431.4	-43.7	.718	9243.3	37811	.0513775
2	etc					37800	
3		etc				37790	
4			etc			37820	
5				etc		37810	
6					etc	37800	

Figure 4.41
REFERRED PARAMETERS

4.5.3.3 UNREFERRED CURVES

Unreferring data allows the calculation of actual specific range fuel flow required at the target W/δ for any temperature condition.

Since specific range is not dependent upon temperature, the program plots specific range versus Mach for standard gross weight at the nominal altitude. The program plots the data for each W/δ singularly, and is capable of combining data similar to figure 4.55.

To calculate actual fuel flow required, the temperature must be specified. The program asks for the temperature entry choice as standard day temperature, delta T_a from standard temperature, or as a specific ambient temperature (T_a). The program plots fuel flow required versus Mach singularly for each W/δ and is capable of combining data similar to figure 4.56.

4.5.3.4 MAXIMUM UNREFERRED VALUES

Maximum values can be selected as in the referred case in paragraph 4.5.3.2.3. Data may appear as:

1.	Pressure altitude	Given in ft MSL
2.	Maximum endurance	Specified at given ˚C, ΔT_a
		Fuel flow in lb/h
		Mach for maximum endurance
3.	Maximum range	Specific range in nmi/lb
		Mach for maximum range

4.5.4 TURBOPROP COMPUTER DATA REDUCTION

Computer programs for range and endurance data reduction for the turboprop are available. As in section 4.5.3, it is assumed detailed instructions on the use of the particular computer program are available. The purpose of the computer data reduction for range and endurance of the turboprop is to automatically calculate and generate referred range and endurance curves for data taken with power set for level flight at any altitude and airspeed. Unreferred curves would also be available given the specific altitude and gross weight desired.

4.5.4.1 DATA ENTRY

From a menu selection the appropriate choice would be made to enter the Range and Endurance program. Data entry requirements for the program would be cued as follows:

1. Basic data:
   a. Type of aircraft (U-21, T-34).
   b. Bureau Number.
   c. Standard gross weight.
   d. Date of tests.
   e. Pilot(s) name(s).
   f. Configuration.
   g. Engine type.
   h. Fuel type.

2. For each data point:
    a. Point number.
    b. Indicated airspeed (kn).
    c. OAT (˚C) or ambient temperature (˚K).
    d. Indicated pressure altitude (ft).
    e. Fuel flow (lb/h).
    f. Torque (psi or ft-lb).
    g. Engine speed.
    h. Gross weight (lb).
    i. Angle of attack (units) if desired and/or available.

Note the following:

1. Basic data is that common to all data points. Item 1.c is used for calculations. The rest of basic data is header information for the final plots.

2. V_o, H_{P_o} and OAT for each point are observed variables. The program would need to know instrument and position error corrections to determine V_c, H_{P_c}, and T_{ic}. The program may require the corrections be made before the data is input.

3. Temperature may be input as either ambient (degrees Kelvin) or OAT (degrees Celsius). The program would be able to use either.

4. In multi-engine aircraft, fuel flow and torque would normally be the total of all engines. The program might be capable of accepting individual engine torque if it is set up to recognize multiple engines.

5. Gross weight would be calculated from the fuel remaining plus the zero fuel weight of the aircraft.

4.5.4.1.1 EQUATIONS USED

Position error calculations from calibrations:

$$H_{P_c} = H_{P_o} + \Delta H_{P_{ic}} + \Delta H_{pos} \qquad \text{(Eq 4.88)}$$

$$V_c = V_o + \Delta V_{ic} + \Delta V_{pos} \qquad \text{(Eq 4.90)}$$

Equivalent airspeed:

$$V_e = \sqrt{\frac{2q}{\rho_{ssl}}} = \sqrt{\frac{\sigma 2q}{\rho_a}} = \sqrt{\sigma}\ V_T \qquad \text{(Eq 4.106)}$$

Pressure ratio:

$$\delta = f\left(H_{P_c}\right) \qquad \text{(Eq 4.99)}$$

If ambient temperature (˚K) was entered:

$$^{\circ}C = {^{\circ}K} - 273.15 \qquad \text{(Eq 4.102)}$$

If OAT was entered:

$$T_a = f\left(OAT\right) \qquad \text{(Eq 4.104)}$$

Temperature ratio:

$$\theta = \frac{T_a}{T_{ssl}} \qquad \text{(Eq 4.38)}$$

Referred fuel flow:

$$\dot{W}_{f_{ref}} = \frac{\dot{W}_f}{\delta \sqrt{\theta}} \qquad \text{(Eq 4.93)}$$

Equivalent shaft horsepower:

$$SHP_e = SHP\sqrt{\sigma} \qquad \text{(Eq 4.107)}$$

Specific fuel consumption::

$$\mathrm{SHPSFC} = \frac{\dot{W}_f}{\mathrm{SHP}} \tag{Eq 4.82}$$

Where:

δ	Pressure ratio	
$\Delta H_{P_{ic}}$	Altimeter instrument correction	ft
ΔH_{pos}	Altimeter position error	ft
ΔV_{ic}	Airspeed instrument correction	kn
ΔV_{pos}	Airspeed position error	kn
H_{P_c}	Calibrated pressure altitude	ft
H_{P_o}	Observed pressure altitude	ft
OAT	Outside air temperature	˚C
P_a	Ambient pressure	psf
q	Dynamic pressure	psf
θ	Temperature ratio	
σ	Density ratio	
SHP	Shaft horsepower	hp
SHP_e	Equivalent shaft horsepower	hp
SHPSFC	Shaft horsepower specific fuel consumption	lb/h/hp
T_a	Ambient temperature	˚C or ˚K
T_{ssl}	Standard seal level temperature	˚C or ˚K
V_c	Calibrated airspeed	kn
V_e	Equivalent airspeed	kn
V_o	Observed airspeed	kn
V_T	True airspeed	kn
W	Weight	lb
W_f	Fuel weight	lb
$\dot{W}_{f\,ref}$	Referred fuel flow	lb/h
$\dot{W}_f$	Fuel flow	lb/h.

4.5.4.2 REFERRED CURVES

4.5.4.2.1 LINEARIZED SHAFT HORSEPOWER

The program produces a plot of equivalent shaft horsepower times equivalent airspeed as a function of equivalent airspeed to the fourth power for a standard gross weight. An essentially linearized plot of the data results for easier interpretation. Using a form of Eq 4.78, the plot appears similar to figure 4.26 with SHP_e replacing THP_e by assuming propeller efficiency (η_P) is 100%. Figure 4.42 illustrates.

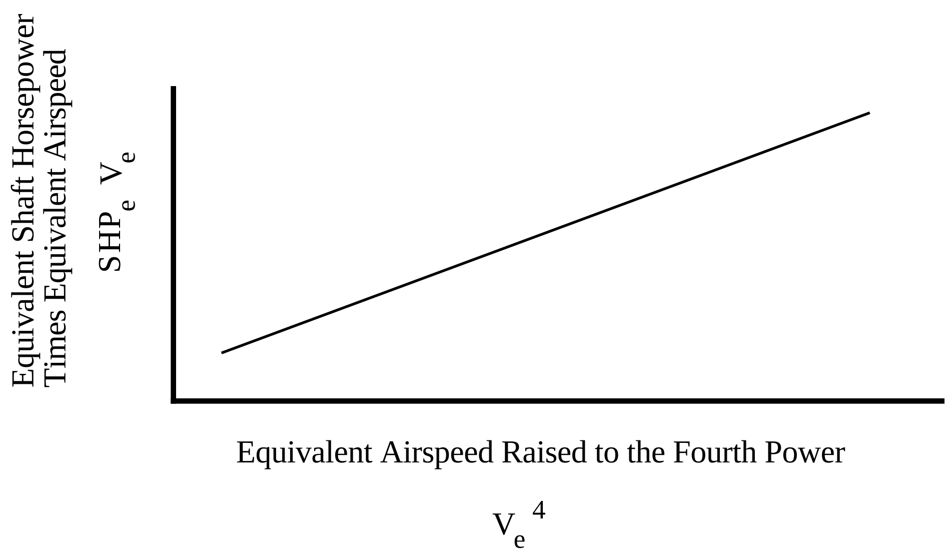

Figure 4.42
LINEARIZED SHAFT HORSEPOWER

4.5.4.2.2 EQUIVALENT SHP

Figure 4.42 is a working plot. Once the curve is fitted through the data points, equivalent SHP versus equivalent airspeed is determined. Fairly simple math converts figure 4.42 to figure 4.43.

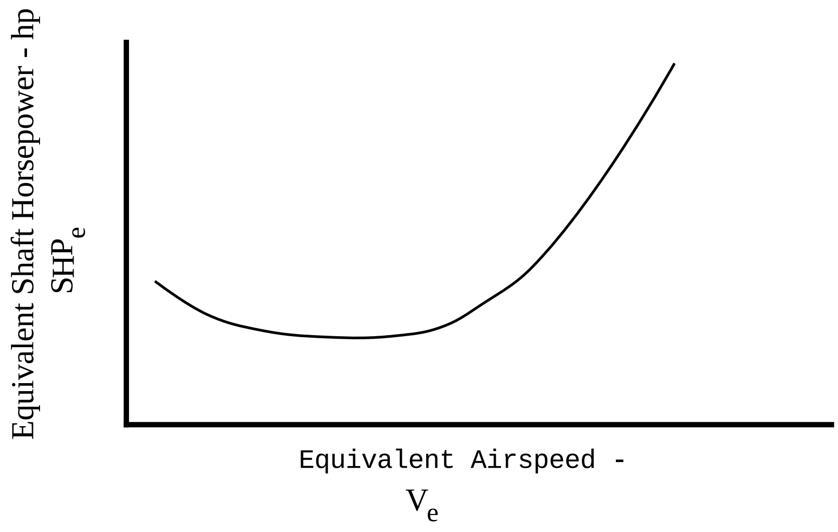

Figure 4.43
EQUIVALENT SHP

4.5.4.2.3 REFERRED FUEL FLOW

The program also calculates referred fuel flow for the data points entered and plots them against the referred shaft horsepower producing a plot similar to figure 4.44. The curve is used later when unreferring fuel flow and presenting the relationship to calibrated airspeed.

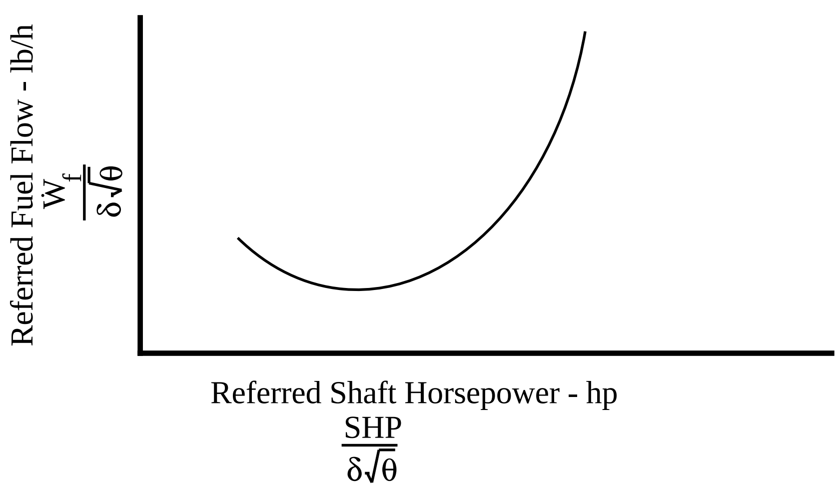

Figure 4.44
REFERRED FUEL FLOW AND SHP

4.5.4.2.4 SPECIFIC FUEL CONSUMPTION

Specific fuel consumption is defined in Eq 4.82 in terms of SHP, that is pounds per hour fuel flow per shaft horsepower.

$$\text{SHPSFC} = \frac{\dot{W}_f}{\text{SHP}} \tag{Eq 4.82}$$

The computer program uses the relationship to calculate and plot figure 4.45 from figure 4.44.

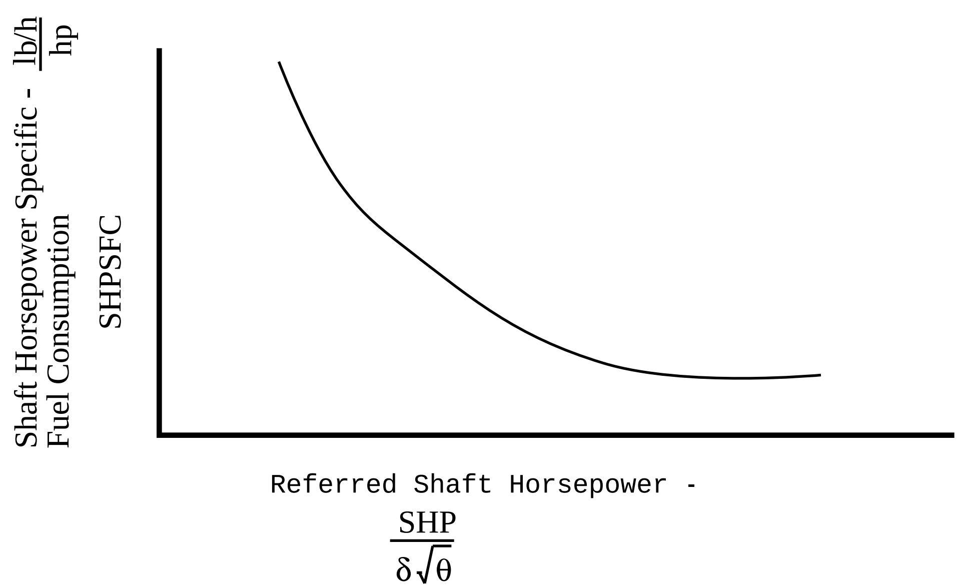

Figure 4.45
SPECIFIC FUEL CONSUMPTION

4.6 DATA ANALYSIS

4.6.1 JET RANGE AND ENDURANCE

Once the referred level flight data is developed, unreferred performance can be obtained for a specified weight, altitude and ambient temperature. The following equations are used.

$$T_a = T_{a_{Std}} + \Delta T_a \qquad \text{(Eq 4.108)}$$

$$\dot{W}_{f_{ref}} = \frac{\dot{W}_f}{\delta\sqrt{\theta}} \qquad \text{(Eq 4.93)}$$

$$S.R. = \frac{a_{ssl}\,M}{\left(\frac{\dot{W}_f}{\delta\sqrt{\theta}}\right)\delta} \qquad \text{(Eq 4.109)}$$

Where:

a_{ssl}	Standard sea level speed of sound	661.483 kn
δ	Pressure ratio	
ΔT_a	Temperature differential	˚C, ˚K
M	Mach number	
θ	Temperature ratio	
S.R.	Specific range	nmi/lb
T_a	Ambient temperature	˚C or ˚K
$T_{a_{Std}}$	Standard ambient temperature	˚C or ˚K
$\dot{W}_f$	Fuel flow	lb/h
$\dot{W}_{f\ ref}$	Referred fuel flow	lb/h.

From figure 4.39 the data is unreferred for a specified gross weight, altitude, and ambient temperature as follows:

Step	Parameter	Notation	Formula	Units	Remarks
1	Referred weight	W/δ		lb	Choose one
2	Weight	W		lb	Select one and H_P is defined
3	Pressure altitude	H_P		ft	Select one and W is defined
4	Ambient temperature	T_a	Eq 4.108	˚C	Standard day from Appendix VI for (3), or for non standard day, or specify
5	Referred fuel flow	$\dot{W}_{f\ ref}$	Eq 4.93	lb/h	From figure 4.39 at each M
6	Actual fuel flow	$\dot{W}_f$		lb/h	Unrefer for δ and θ at (1) or (2)
7	Specific Range	S.R.	Eq 4.109	nmi/lb	

Plot unreferred fuel flow, $\dot{W}_f$, versus M for a specified gross weight, altitude, and ambient temperature; and plot S.R. versus M for the same gross weight, altitude, and ambient temperature as in figure 4.46.

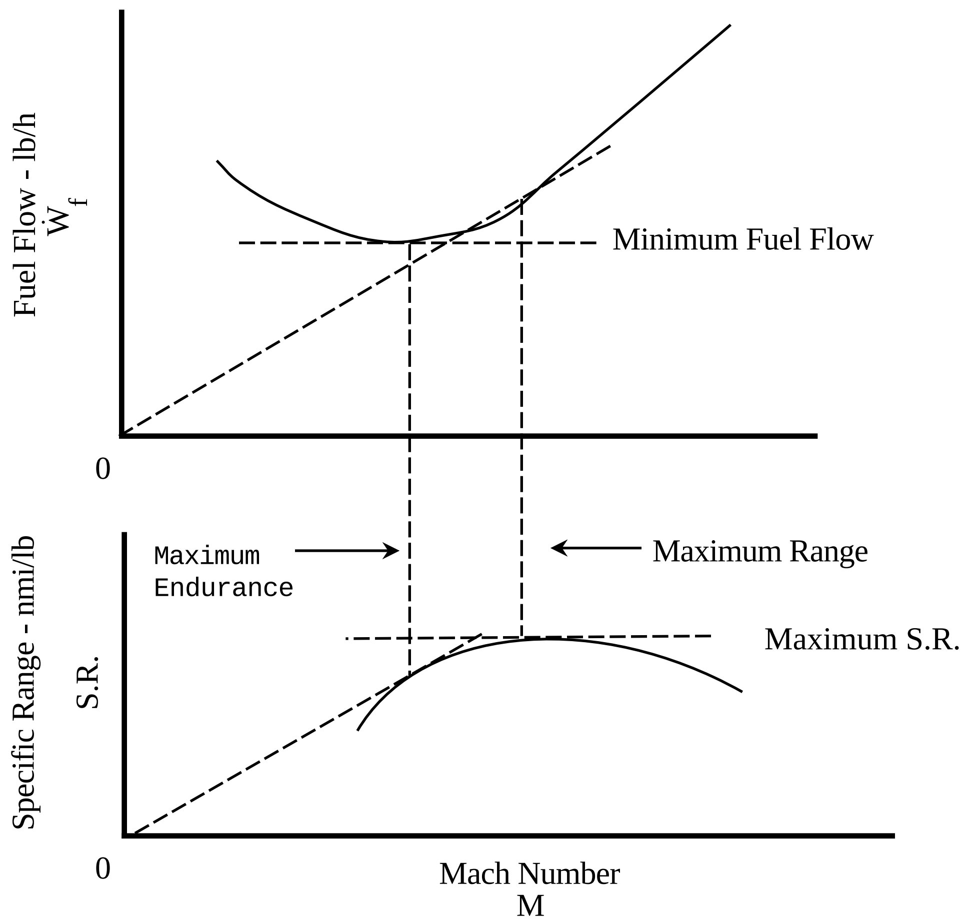

Figure 4.46
UNREFERRED LEVEL FLIGHT PERFORMANCE

As previously shown in figure 4.17, maximum specific range occurs at the tangent to the fuel flow versus V_T curve of a line drawn from the origin. Since V_T =K x M for a given T_a, the same conclusion applies to figure 4.46. Maximum endurance Mach (minimum fuel flow) occurs at the tangent to the specific range versus Mach curve of a line drawn from the origin in figure 4.46.

4.6.2 NON STANDARD DAY RANGE AND ENDURANCE

The referred data curve, figure 4.39, can be generated from test day data without regard to the ambient temperature. For a given altitude and gross weight, an increase in ambient temperature results in an increased actual fuel flow so the ratio $\frac{\dot{W}_f}{\sqrt{\theta}}$ remains constant. Figure 4.47 is figure 4.46 plotted versus V_T for standard and hot days.

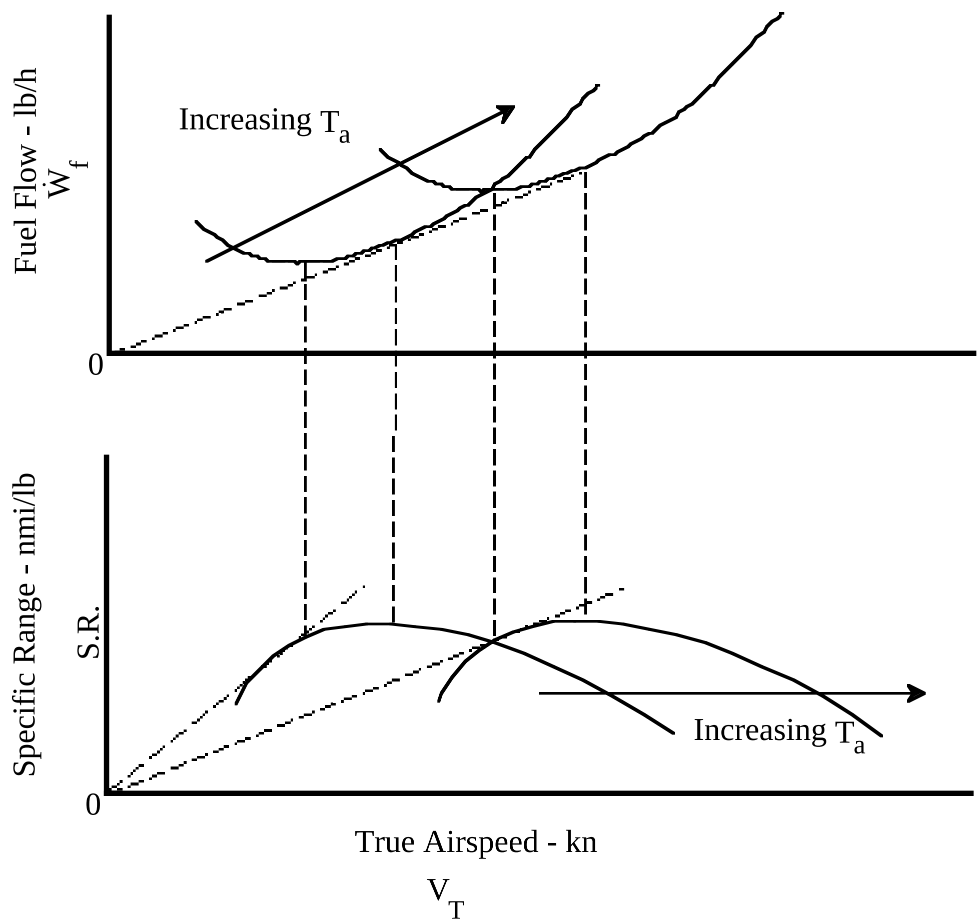

Figure 4.47
LEVEL FLIGHT NON STANDARD DAY

Regardless of ambient temperature, the aircraft specific range is constant. On a hot day the actual fuel flow increases so $\frac{\dot{W}_f}{\sqrt{\theta}}$ is constant. V_T is scaled up by the same factor. To illustrate:

$$V_{T_{Hot\ day}} = 661.483\ M\ \sqrt{\theta_{Std}}\left(\frac{\sqrt{\theta_{Hot\ day}}}{\sqrt{\theta_{Std}}}\right) \qquad \text{(Eq 4.110)}$$

$$\dot{W}_{f_{Hot\ day}} = \dot{W}_{f_{Std}}\left(\frac{\sqrt{\theta_{Hot\ day}}}{\sqrt{\theta_{Std}}}\right) \qquad \text{(Eq 4.111)}$$

from:

$$\frac{\dot{W}_f}{\sqrt{\theta}} = ¢ \qquad \text{(Eq 4.112)}$$

Where:

M	Mach number	
θ	Temperature ratio	
V_T	True airspeed	kn
$\dot{W}_f$	Fuel flow	lb/h
¢	Constant.	

As ambient temperature increases, endurance time decreases since $\dot{W}_f$ increases to keep $\frac{\dot{W}_f}{\sqrt{\theta}}$ constant.

4.6.3 WIND EFFECTS ON RANGE AND ENDURANCE

The effect of wind on range is very important. A head wind reduces range and a tail wind increases range. Specific range for no wind was $\frac{V_T}{\dot{W}_f}$. For a head wind or tail wind, ground speed for the fuel flow needs to be maximized $\frac{GS}{\dot{W}_f}$. Figure 4.48 shows the variation with speed to maximize specific range in the presence of wind.

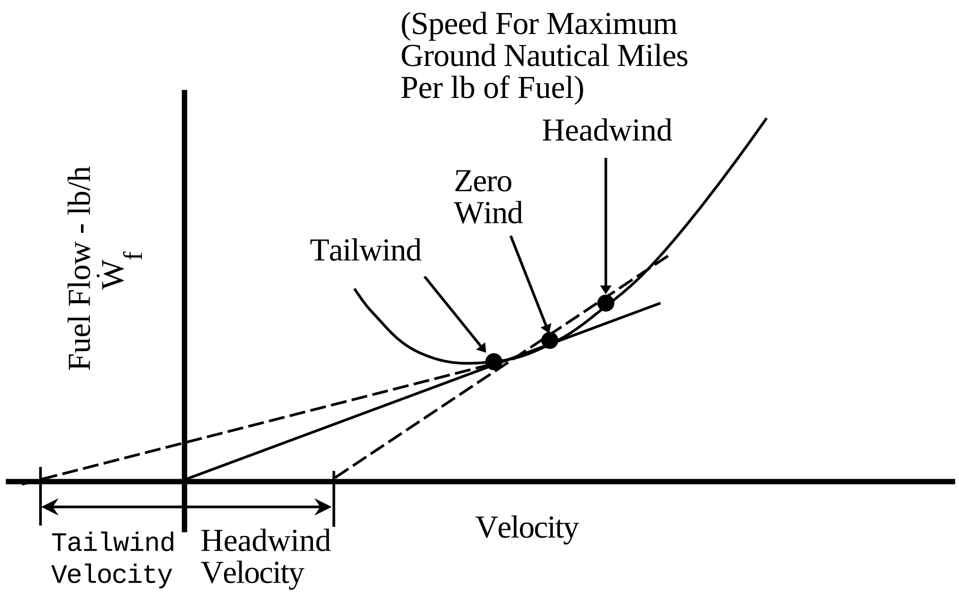

Figure 4.48
WIND EFFECT ON LEVEL FLIGHT RANGE

With zero wind, a line from the origin tangent to the curve locates the maximum range condition. With a head wind, the speed for maximum ground range is located by a line drawn tangent from a velocity offset equal to the head wind. The range is at some higher velocity and fuel flow. Although the range is less than when at zero wind, the higher velocity minimizes the range loss due to the head wind. The effect of a tail wind is the reverse. The correction in speed to account for wind becomes critical only when wind velocities exceed 25 percent of the true airspeed.

4.6.4 RANGE AND ENDURANCE PROFILES

The foregoing analysis presented the data for a single W/δ. To determine the actual range and endurance, data from all W/δ flights are combined.

4.6.4.1 REFERRED FUEL FLOW COMPOSITE

When all W/δ flights are combined, the results look like figure 4.49 for lines of constant W/δ from minimum to maximum.

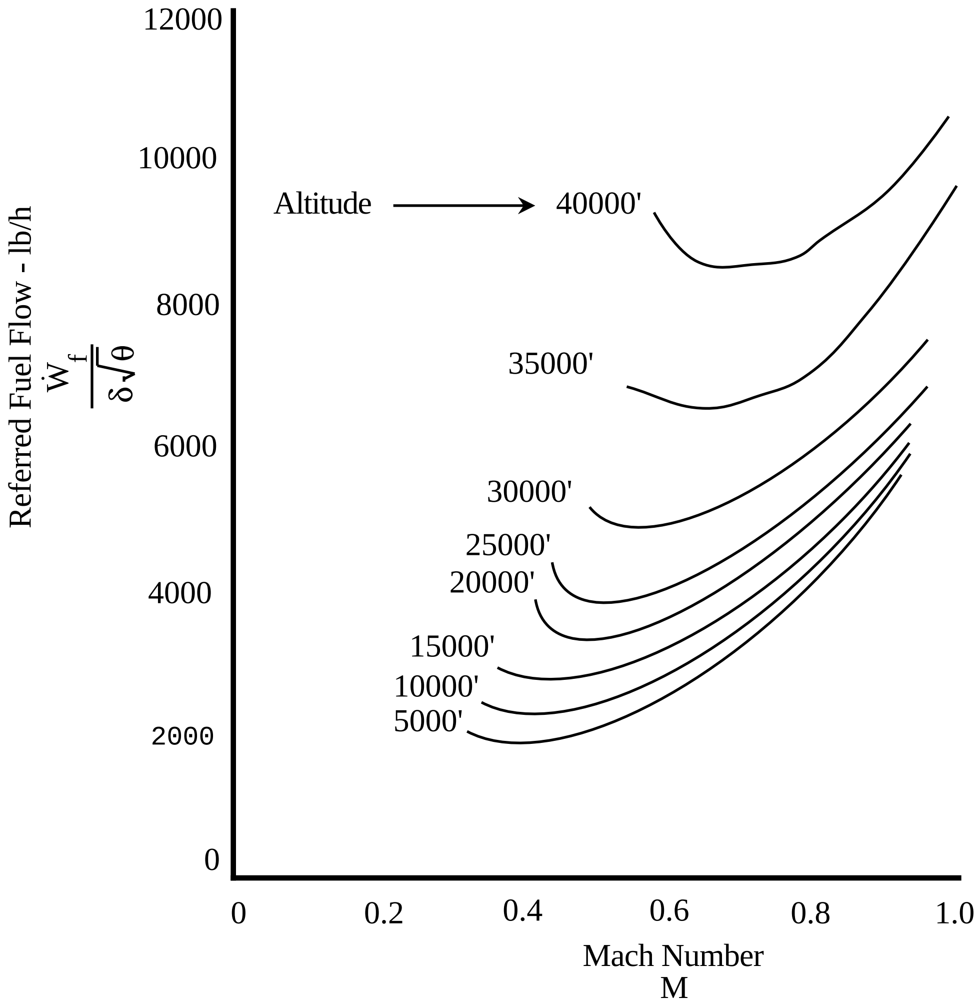

Figure 4.49
REFERRED FUEL FLOW COMPOSITE

4.6.4.2 REFERRED SPECIFIC RANGE COMPOSITE

Range performance can be determined by rearranging figure 4.49 into another referred composite curve called referred specific range as shown in figure 4.50.

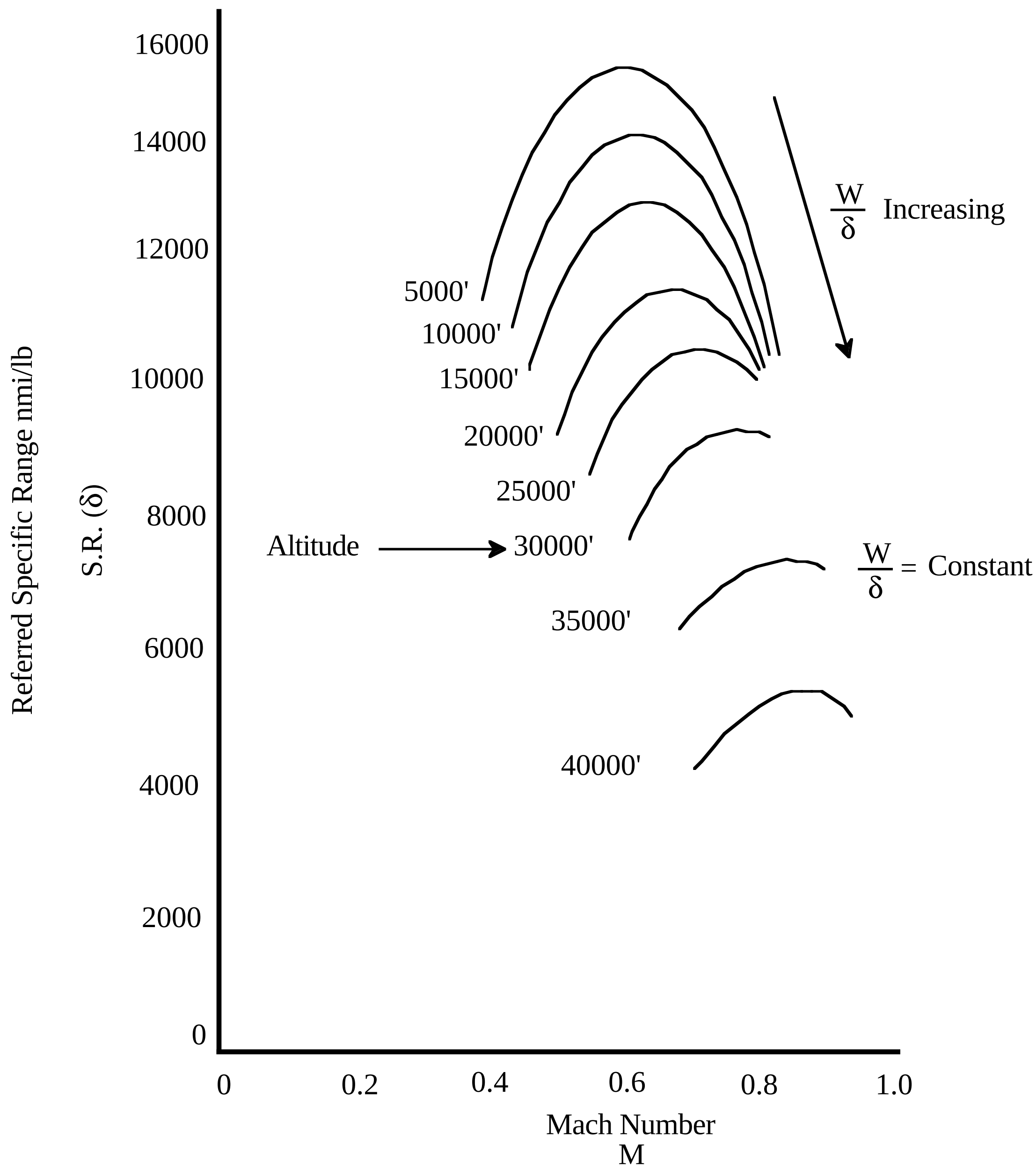

Figure 4.50
REFERRED SPECIFIC RANGE

Equations for the referred specific range determination are:

$$\text{S.R.} = \frac{a_{ssl}\,M}{\left(\dfrac{\dot{W}_f}{\delta\sqrt{\theta}}\right)\delta} \qquad \text{(Eq 4.109)}$$

where referred specific range is defined as the product of S.R. and δ.

$$S.R.\,\delta = \frac{661.483M}{\left(\dfrac{\dot{W}_f}{\delta\sqrt{\theta}}\right)}$$

(Eq 4.105)

Where:

a_{ssl}	Standard sea level speed of sound	661.483 kn
δ	Pressure ratio	
M	Mach number	
θ	Temperature ratio	
S.R.	Specific range	nmi/lb
$\dot{W}_f$	Fuel flow	lb/h.

Specific range is calculated by Eq 4.109 as in paragraph 4.6.1 for each W/δ line then multiplied by the pressure ratio δ corresponding to the altitudes chosen to obtain referred specific range.

4.6.4.2.1 RANGE CALCULATION

When the specific range is known, the actual range is found by integrating the specific range over the fuel burned. Since the gross weight decreases as fuel is consumed, the specific range is constantly changing. Evaluating how the specific range changes depends on the type of cruise control being used. The cruise may be flown at some schedule of altitude, Mach, fuel flow, engine speed, cruise climb, etc. with each type resulting in different range performance. Determining the optimum range profile is the goal.

The following equations show the development of the range equation in terms of the data actually measured in flight tests:

$$\text{Range} = \int_0^{W_{f_{Used}}} (S.R.)\, dW_f$$

(Eq 4.113)

Fuel used is just the change in gross weight, so:

$$\text{Range} = \int_{W_1}^{W_2} (\text{S.R.})\, dW \qquad \text{(Eq 4.114)}$$

Converting the quantities inside the integral into terms of referred parameters (S.R. x δ) and multiplying top and bottom by W, the range is a function of referred parameters (S.R. x δ) and (W/δ):

$$\text{Range} = \int_{W_1}^{W_2} (\text{S.R. } \delta) \left(\frac{W}{\delta}\right) \frac{1}{W}\, dW \qquad \text{(Eq 4.115)}$$

In terms of measured data:

$$\text{Range} = \int_{W_1}^{W_2} \left[\frac{661.483\ M}{\left(\frac{\dot{W}_f}{\delta \sqrt{\theta}}\right)} \frac{W}{\delta} \right] \frac{dW}{W} \qquad \text{(Eq 4.116)}$$

Where:

δ	Pressure ratio	
M	Mach number	
θ	Temperature ratio	
S.R.	Specific range	nmi/lb
W	Weight	lb
W_1	Initial cruise weight	lb
W_2	Final cruise weight	lb
W_f	Fuel weight	lb
$\dot{W}_f$	Fuel flow	lb/h.

4.6.4.2.2 INTERPOLATING THE REFERRED DATA CURVES

The W/δ flights were planned to include W/δ ranging from minimum to maximum values to make extrapolation of the referred curves unnecessary. Since the number of test flights are finite, interpolation is necessary. However, interpolation between the lines of constant W/δ is not linear. Interpolation uses the following equations and resultant figures:

$$\frac{D}{\delta} = f\left(M, \frac{W}{\delta}\right) \qquad \text{(Eq 4.27)}$$

For a parabolic drag polar :

$$\frac{D}{\delta} = \frac{C_{D_{P(M)}} \gamma P_{ssl} M^2 S}{2} + \frac{2\left(W/\delta\right)^2}{\pi e_{(M)} AR\, S\, \gamma P_{ssl} M^2} \qquad \text{(Eq 4.26)}$$

Interpolating between W/δ at constant M and substituting for all constants in Eq 4.26, Eq 4.117 results and figure 4.51 can be developed.

$$\frac{D}{\delta} = K_3 + K_4 \left(\frac{W}{\delta}\right)^2 \qquad \text{(Eq 4.117)}$$

Where:

AR	Aspect ratio	
$C_{D_{p\,(M)}}$	Parasite drag coefficient at high Mach	
D	Drag	lb
δ	Pressure ratio	
e	Oswald’s efficiency factor	
γ	Ratio of specific heats	
K_3	Constant	
K_4	Constant	
M	Mach number	
π	Constant	
P_{ssl}	Standard sea level pressure	2116.217 psf
S	Wing area	ft^2
W	Weight	lb.

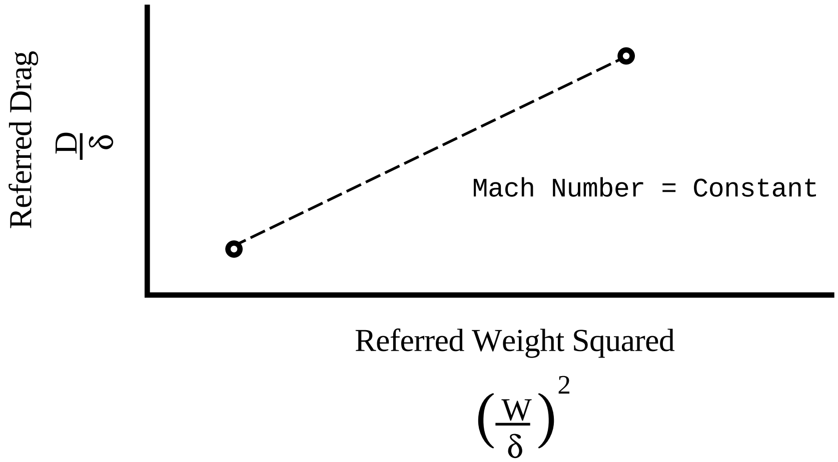

Figure 4.51
INTERPOLATED W/δ AT CONSTANT MACH

Since D/δ is proportional to referred fuel flow, $\frac{\dot{W}_f}{\delta\sqrt{\theta}}$, figure 4.52 can be develop in similar fashion.

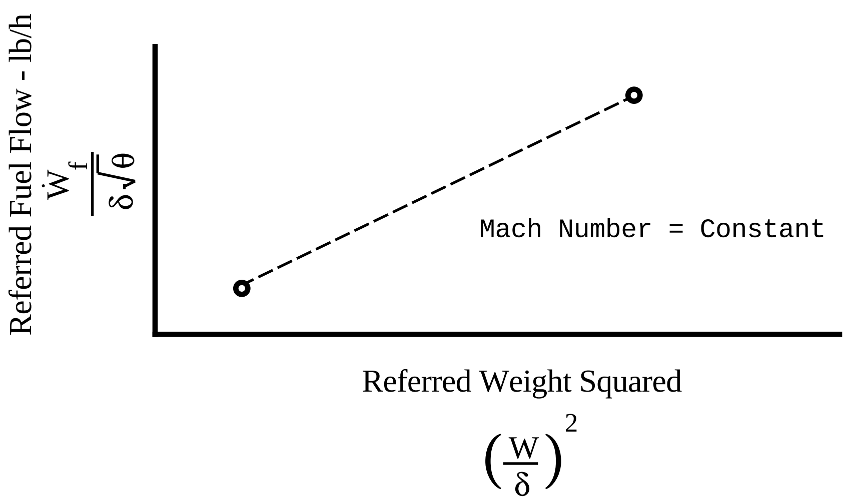

Figure 4.52
INTERPOLATED W/δ ALTERNATE AT CONSTANT MACH

For an aircraft exhibiting a cubic drag polar such as the T-38, figure 4.53 works for interpolation.

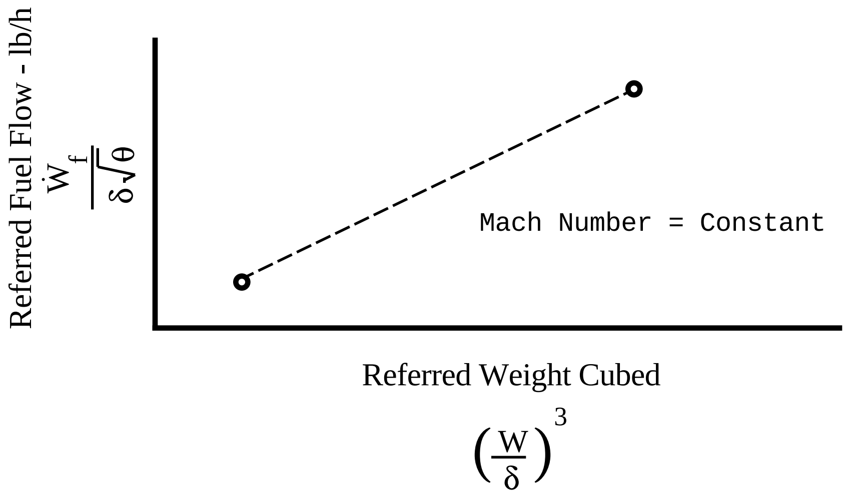

Figure 4.53
CUBIC DRAG POLAR INTERPOLATED W/δ AT CONSTANT MACH

4.6.4.3 OPTIMUM RANGE

From Eq 4.105, the product of referred specific range and referred weight is the range factor (R.F.) and is useful in determining optimum range.

$$\text{R.F.} = \left[\left(\text{S.R. } \delta\right) \frac{W}{\delta} \right] \tag{Eq 4.118}$$

Therefore:

$$\text{R.F.} = \left[\frac{661.483 \text{ M}}{\left(\frac{\dot{W}_f}{\delta\sqrt{\theta}}\right)} \left(\frac{W}{\delta}\right) \right] \tag{Eq 4.119}$$

$$\text{Range} = \int_{W_1}^{W_2} \left(\text{R.F.}\right) \frac{dW}{W} \tag{Eq 4.120}$$

Where:

δ	Pressure ratio	
M	Mach number	
θ	Temperature ratio	
R.F.	Range factor	
S.R.	Specific range	nmi/lb
W	Weight	lb
$\dot{W}_f$	Fuel flow	lb/h.

Optimum range performance then is achieved by maximizing the range factor. In other words, if the range factor is held at its maximum value, maximum aircraft range results.

4.6.4.3.1 CALCULATING MAXIMUM RANGE FACTOR AND OPTIMUM MACH

Maximum range factor is calculated from figure 4.50 by multiplying the maximum of each referred specific range curve by its associated W/δ. The calculated values result in the maximum range factor (Eq 4.118) and the Mach number for optimum range factor for each W/δ curve. These values can be plotted as in figure 4.54.

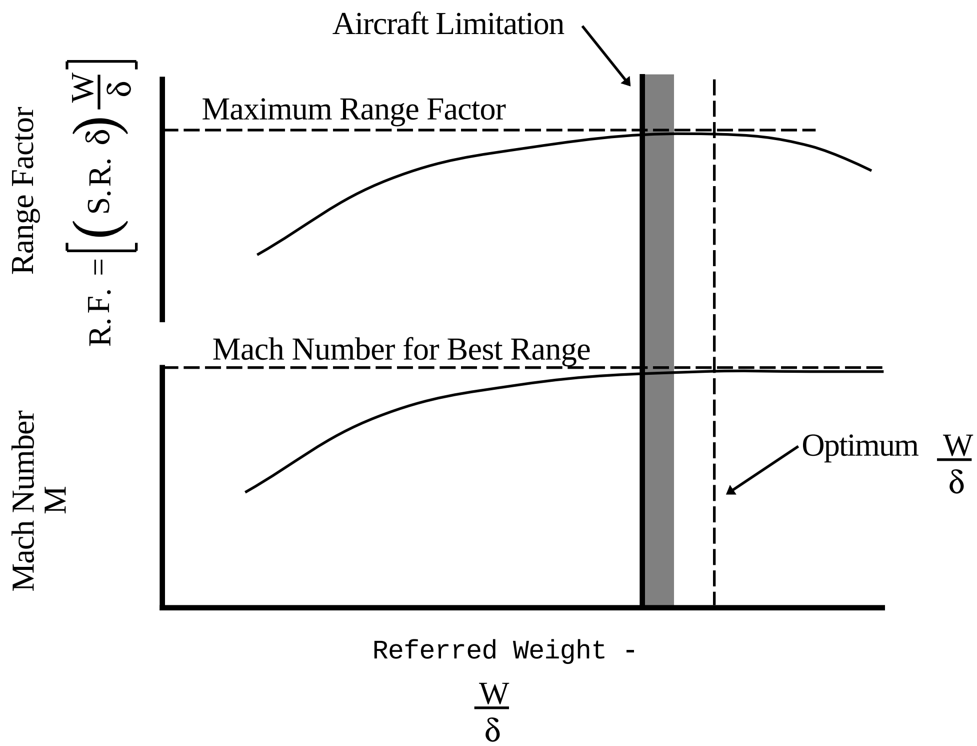

Figure 4.54
OPTIMUM RANGE

The peak range factor may be unobtainable if:

1. The optimum W/δ is in excess of the absolute ceiling of the aircraft.
2. Safety or operational considerations preclude ascending to these high altitudes.

In these cases the maximum range factor and optimum Mach occur at the highest W/δ tested.

If able to fly high enough, the results appear as in figure 4.55.

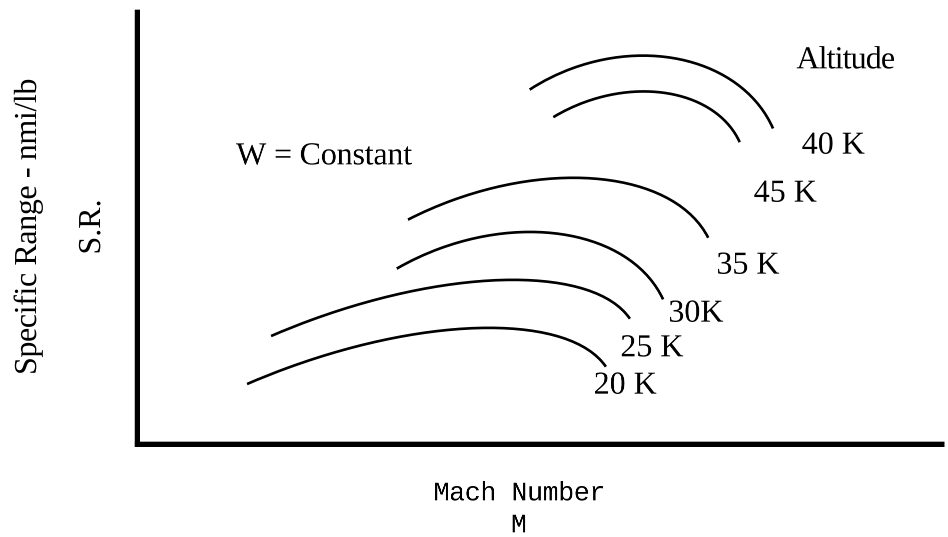

Figure 4.55
ACTUAL SPECIFIC RANGE

The range factor increases with increasing W/δ (altitude for a given weight) and ultimately reaches a maximum. In figure 4.55, the Mach number for best range is also determined once the optimum W/δ is known.

Actual specific range can be determined by unreferring figure 4.54 for a specific weight as a function of Mach number and pressure altitude as in figure 4.55. Figure 4.55 shows the particular airspeed and altitude to fly for maximum S.R.

4.6.4.4 OPTIMUM ENDURANCE FOR JETS

Fuel flow is proportional to the thrust. Therefore, a jet achieves maximum specific endurance when operated at minimum thrust required or where the lift to drag ratio is maximum, $\left.\frac{L}{D}\right|_{max}$. In subsonic flight below the drag rise Mach, $\left.\frac{L}{D}\right|_{max}$ occurs at a specific value of lift coefficient where the ratio of lift to drag coefficients is maximum, $\left.\frac{C_L}{C_D}\right|_{max}$. For a given aircraft this ratio is independent of weight or altitude. If the aircraft of a given weight and configuration is operated at various altitudes, the value of the minimum thrust required is unaffected as discussed in paragraph 4.3.4.3. Endurance is really only a function of engine performance.

Specific fuel consumption of the jet engine is strongly affected by operating engine speed and altitude. The jet engine generally operates more efficiently near normal rated engine speed and the low temperatures of the stratosphere to produce low specific fuel consumption. Increased altitude provides the favorable lower inlet air temperature and requires a greater engine speed to provide the thrust required at $\left.\frac{C_L}{C_D}\right|_{max}$. The typical jet experiences an increase in specific endurance with altitude with the peak values occurring at or near the tropopause. An example is the single engine jet with a maximum specific endurance at 35,000 feet which is at least 40% greater than the maximum value at sea level. If the jet is at low altitude and it is necessary to hold for a considerable time, maximum time in the air is obtained by beginning a climb to some optimum altitude dependent upon the fuel quantity available. The fuel expended during the climb may be offset by lower TSFC at the higher altitude providing greater total endurance.

The altitude where TSFC is minimum is called the critical altitude. At altitudes above critical, the TSFC increases resulting in decreased endurance performance. Decreasing engine component efficiencies caused by the reduced Reynold's number and operating engine speed above the design value encountered at very high altitudes lead to the increased TSFC. For present day engines, the critical altitude usually occurs fairly close to the beginning of the isothermal layer near 36,000 feet.

Endurance considerations discussed above all depend upon steady level flight at a particular altitude. Fuel required to climb to the altitude for optimum level flight endurance must be considered in the overall endurance performance. Many times it is more advantageous for overall endurance to remain at a lower altitude than to expend fuel for climbing. By compiling climb fuel consumption data with level flight endurance data, guidelines can be established in making decisions to climb or maintain altitude.

4.6.4.4.1 UNREFERRED ENDURANCE

A referred fuel flow composite is presented in figure 4.49. The data can be unreferred and replotted as in figure 4.56 as actual fuel flow curves for various altitudes at a given weight and configuration. The calibrated airspeed for minimum fuel flow is found to increase some with increasing altitude. The figure is based on a specific temperature at each altitude. Hot day endurance requires higher fuel flow than on a standard day since for a given weight, altitude, and airspeed, $\frac{\dot{W}_f}{\sqrt{\theta}}$ is constant and not $\dot{W}_f$.

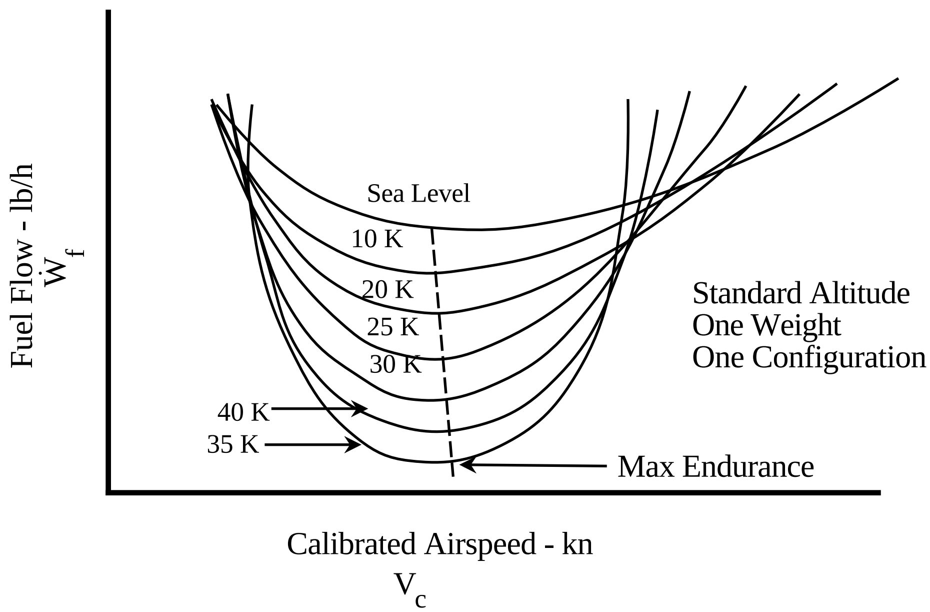

Figure 4.56
ACTUAL FUEL FLOW

Once the target airspeed for maximum endurance is known, the fuel flow and optimum altitude for a given gross weight can be found from a composite plot of minimum fuel flow points. Figure 4.57 illustrates.

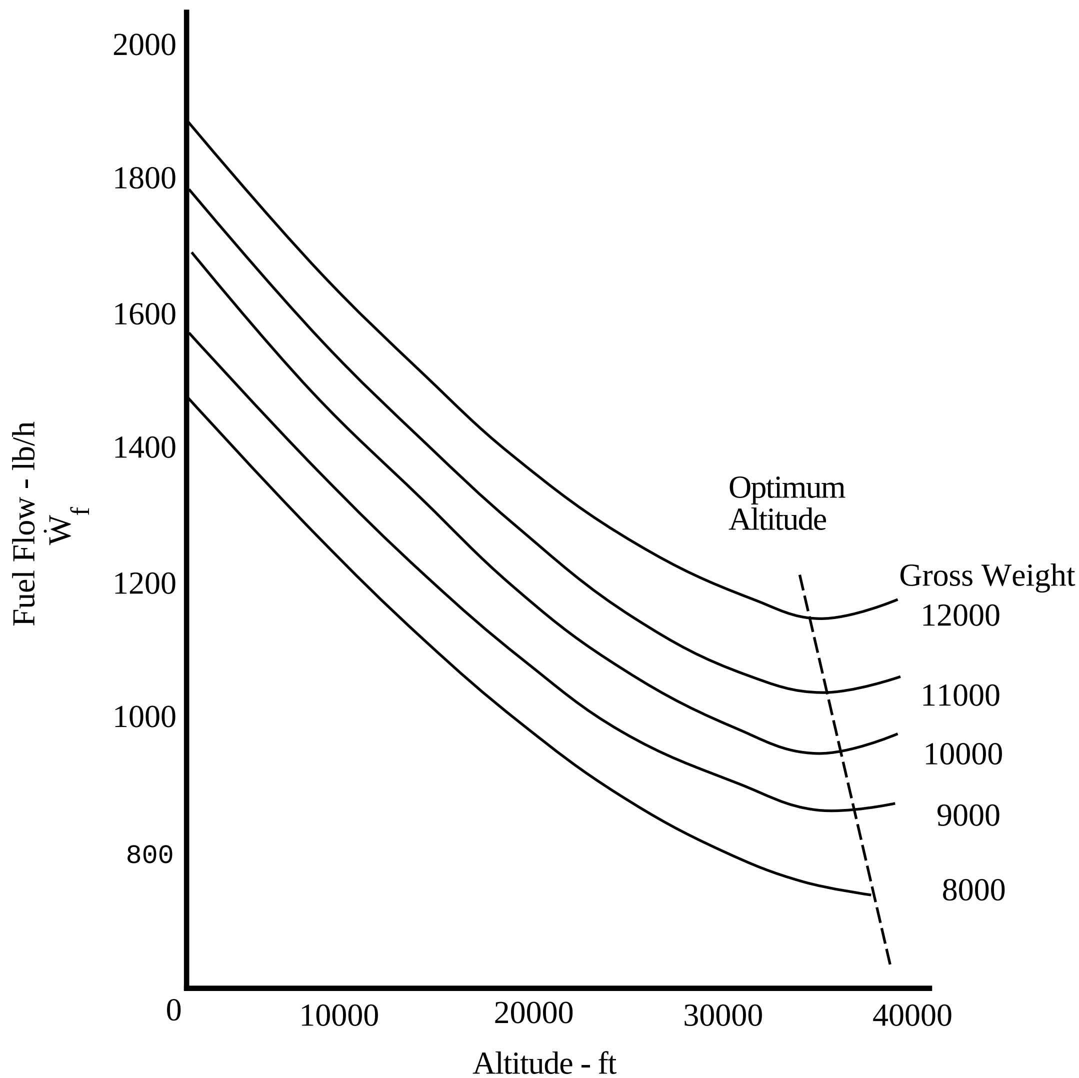

Figure 4.57
OPTIMUM ENDURANCE

4.6.5 FERRY RANGE

Total range capability was determined in paragraph 4.5.2. To summarize, the total range capability of the aircraft included distance traveled during climb plus distance traveled during cruise. Range credit is not allowed for takeoff, acceleration to climb speed, and descent. Distance traveled during the climb is obtained from the climb tests and distance traveled during cruise is computed by using the range factor and Eq 4.120 where W_1 was the initial cruise weight (end of climb) using fuel allowances for ground time and acceleration to climb speed and fuel for climb from the requirements. W_2 was the final cruise weight at the end of the cruise phase. At each test point, altitude, airspeed, weight, ambient temperature, engine speed, and Mach number were recorded while flying a pre-planned profile, including a slow climb to optimize range. Figure 4.58 presents typical format for ferry range data.

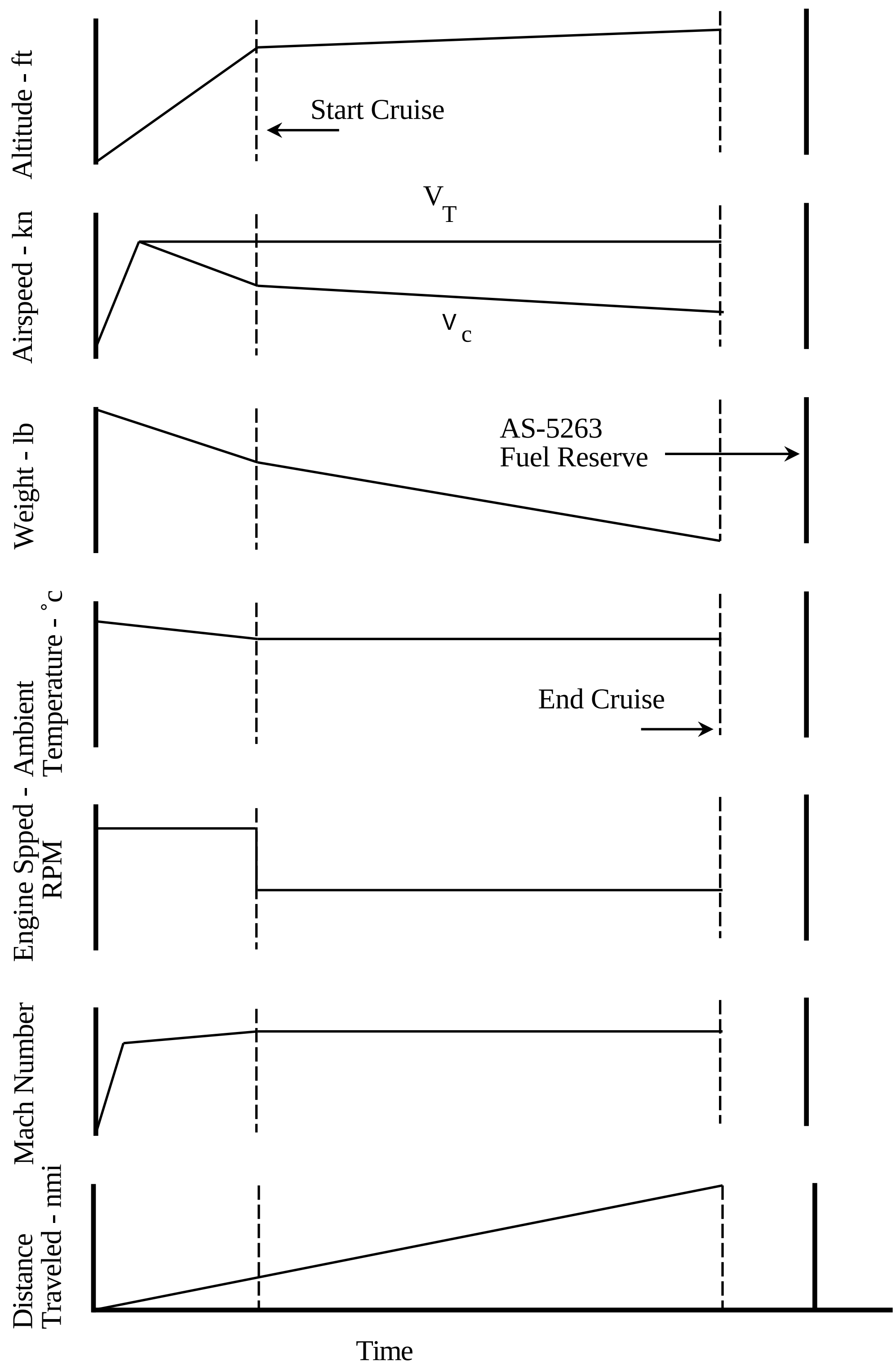

Figure 4.58

FERRY RANGE

4.6.6 CRUISE CLIMB AND CONTROL

Cruise climb and control implies that an airplane is operated to maintain the recommended long range cruise condition throughout the flight. Since fuel is consumed during cruise, the gross weight of the airplane will vary and optimum airspeed, altitude, and power setting can vary. Cruise control means the control of optimum airspeed, altitude, and power setting to maintain the maximum specific range condition. At the beginning of cruise, the high initial weight of the airplane will require specific values of airspeed, altitude, and power setting to produce the recommended cruise condition. As fuel is consumed and the airplane gross weight decreases, the optimum airspeed and power setting may decrease or the optimum altitude may increase. In addition, the optimum specific range will also increase. The pilot must use the proper cruise control technique to ensure that the optimum conditions are maintained.

Cruise altitude of the turbojet should be as high as possible within compressibility or thrust limits. In general, the optimum altitude to begin cruise is the highest altitude at which the maximum continuous thrust can provide the optimum aerodynamic conditions. The optimum altitude is determined mainly by the gross weight at the beginning of cruise. For the majority of jets this altitude will be at or above the tropopause for normal cruise configurations.

Most jets which have transonic or moderate supersonic performance will obtain maximum range with a high subsonic cruise. However, an airplane designed specifically for high supersonic performance will obtain maximum range with a supersonic cruise; subsonic operation will cause low lift to drag ratios, poor inlet and engine performance and reduce the range capability.

The cruise control of the jet is considerably different from that of the prop airplane. Since the specific range is so greatly affected by altitude, the optimum altitude for beginning of cruise should be attained as rapidly as is consistent with climb fuel requirements. The range climb schedule varies considerably between airplanes, and the performance section of the flight handbook will specify the appropriate procedure. The descent from cruise altitude will employ essentially the same procedure. A rapid descent is necessary to minimize the time at low altitudes where specific range is low and fuel flow is high for a given engine speed.

During cruise flight of the jet, the gross weight decrease by fuel used can result in two types of cruise control. During a constant altitude cruise, a reduction in gross weight will require a reduction of airspeed and engine thrust to maintain the optimum lift coefficient of subsonic cruise. While such a cruise may be necessary to conform to the flow of traffic, it constitutes a certain operational inefficiency. If the airplane were not restrained to a particular altitude, maintaining the same lift coefficient and engine speed would allow the airplane to climb as the gross weight decreases. Since altitude generally produces a beneficial effect on range, the climbing cruise implies a more efficient flight path.

The cruising flight of the jet will begin usually at or above the tropopause in order to provide optimum range conditions. If flight is conducted at $(C_L^{1/2}/C_D)_{max}$, optimum range will be obtained at specific values of lift coefficient and drag coefficient. When the airplane is fixed at these values of C_L and C_D, and V_T is held constant, both lift and drag are directly proportional to the density ratio σ. Also, above the tropopause, the thrust is proportional to σ when V_T and engine speed are constant. Then a reduction of gross weight by the use of fuel would allow the jet to climb but the jet would remain in equilibrium because lift, drag, and thrust all vary in the same fashion.

The relationship of lift, drag, and thrust is convenient since it justified the condition of a constant velocity. Above the tropopause, the speed of sound is constant and a constant velocity during the cruise climb would produce a constant Mach number. In this case, the optimum values of $(C_L^{1/2}/C_D)$, C_L, and C_D do not vary during the climb since the Mach number is constant. The specific fuel consumption is initially constant above the tropopause but begins to increase at altitudes much above the tropopause. If the specific fuel consumption is assumed to be constant during the cruise climb, a 10% decrease in gross weight from the consumption of fuel would create:

1. No change in Mach number or V_T.
2. A 5% decrease in V_e.
3. A 10% decrease in σ (higher altitude).
4. A 10% decrease in fuel flow.
5. An 11% increase in specific range.

4.6.6.1 CRUISE CLIMB AND CONTROL SCHEDULES

The term cruise climb applies to various types of flight programs designed to improve overall maximum range for a given fuel load. The example in this section refers to a flight programmed to maintain a constant M and W/δ.

To determine how a schedule of constant M and W/δ is flown, consider all factors which are constant when M and W/δ are constant. From the lift equation:

$$C_L = \frac{2\,W}{\gamma\,P_a\,M^2\,S} = \frac{2\,\frac{W}{\delta}}{\gamma\,M^2\,S\,P_{ssl}} \qquad \text{(Eq 4.121)}$$

Since γ, P_{ssl}, and S are constant:

$$C_L = f\left(\frac{W}{\delta}, M^2\right) \qquad \text{(Eq 4.122)}$$

If in a cruise climb, W/δ and M are constant then C_L = Constant.

For steady, level flight, neglecting viscosity:

$$\frac{D}{\delta} = f\left(M, \frac{W}{\delta}\right) \qquad \text{(Eq 4.27)}$$

Therefore D/δ must be constant. The ratio of two constants must also be constant so:

$$\frac{\frac{W}{\delta}}{\frac{D}{\delta}} = \frac{W}{D} = \text{Constant} \qquad \text{(Eq 4.123)}$$

Since W = L in steady level flight, L/D = Constant. Neglecting viscosity, for a constant area engine the following parameters are a function of Mach number and W/δ and are therefore constant:

1. $N/\sqrt{\theta}$ constant referred engine speed.

2.	T_G/δ	constant gross thrust/pressure ratio.
3.	P_{T_6}/δ	constant exit pressure/pressure ratio.
4.	$\dot{W}_f/\delta\sqrt{\theta}$	constant referred fuel flow.

Previously, the specific range parameter was also constant therefore:

$$\frac{661.483\ M}{\left(\frac{\dot{W}_f}{\delta\sqrt{\theta}}\right)} = \text{Constant} \tag{Eq 4.124}$$

From Eq 4.48 and the fact that $\dot{W}_f/\delta\sqrt{\theta}$ and T_{N_x}/δ are constant it can be shown that:

$$\frac{TSFC}{\sqrt{\theta}} = \text{Constant} \tag{Eq 4.125}$$

Where:

C_L	Lift coefficient	
D	Drag	lb
δ	Pressure ratio	
γ	Ratio of specific heats	
M	Mach number	
P_a	Ambient pressure	psf
P_{ssl}	Standard sea level pressure	2116.217 psf
θ	Temperature ratio	
S	Wing area	ft^2
TSFC	Thrust specific fuel consumption	$\frac{lb/h}{lb}$
W	Weight	lb
$\dot{W}_f$	Referred fuel flow	lb/h.

The example can continue further but there are very few constant parameters which are easily measured or available to record. The Mach number, α, H_{P_i}, engine speed, fuel remaining, T_G, and T_G/δ can be read from cockpit instruments so a pilot would be able to fly the following type schedules:

1. Constant α and Mach.
2. Constant W/δ and Mach.
3. Constant T_G/δ and Mach.
4. Constant $N/\sqrt{\theta}$ and Mach.
5. Constant T_G and $N/\sqrt{\theta}$ in an isothermal layer.
6. Other variations or combinations of the above.

Flight testing has indicated that schedule 2 produces the best results with the instrumentation available. A slight modification of schedule 4 with schedule 2 was also recommended as suitable. For variable area engines schedule 3 appeared to have the greatest promise.

Increases in cruising ranges of from 5-6% can be realized using cruise climb techniques as compared to level flight procedures. Of the two factors, altitude and Mach, Mach is the most critical and must be maintained on the assigned value.

4.6.7 RANGE DETERMINATION FOR NON-OPTIMUM CRUISE

Various forms of cruise control appear on the referred specific range curve as shown in figure 4.59.

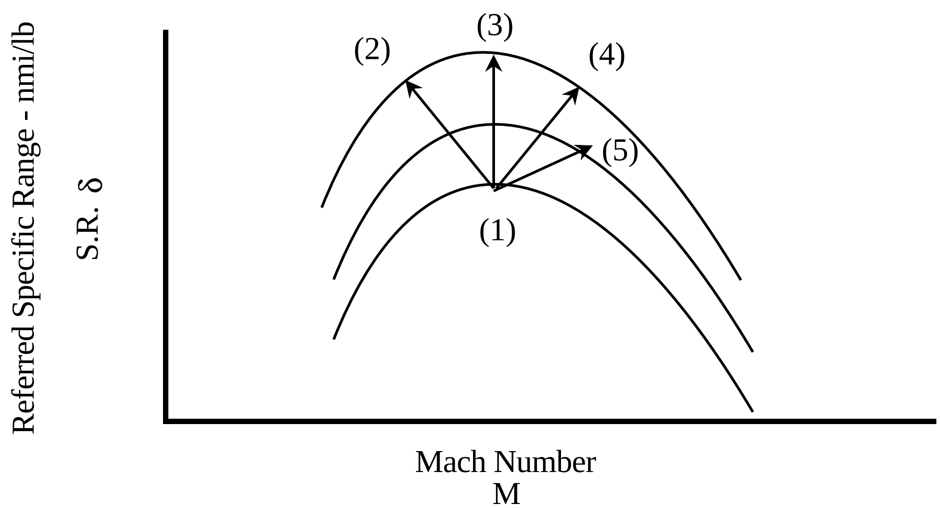

Figure 4.59

CRUISE CONTROL VARIATIONS ON REFERRED SPECIFIC RANGE

Schedule (1) is the cruise climb. The schedule is flown at constant W/δ and M and is characterized by an increase in S.R. and by always remaining at the optimum Mach number. The schedule is a single point on the figure.

Schedule (2) maintains constant altitude and decreases Mach as W/δ decreases to remain on the peaks of the W/δ curves. Mach is always optimum and S.R. increases but not to the same extent as in Schedule (1).

Schedule (3) maintains constant altitude and constant Mach, and as W/δ decreases, the Mach moves away from the optimum. This schedule may not be too far off from schedule (2).

Schedule (4) maintains constant altitude and power setting (either fuel flow or engine speed). A constant altitude W/δ will decrease as W decreases, and at constant power setting, M will increase as W decreases. Mach will not be optimum for max specific range even though S.R. may increase.

Schedule (5) holds constant power setting and calibrated airspeed. As W decreases it will be necessary to climb to maintain constant V_c. Climbing at constant V_c will yield an increase in Mach number which moves the point away from Mach as W/δ decreases. S.R. will increase with decreased W, but the increase will not be as great as the other schedules.

Some of these schedules are simpler than others to use. For example, it is easier to fly constant altitude and power setting than to climb at constant M and W/δ. However, experience has proven that savings in fuel or increase in range can be attained by executing an optimum climb covered in Chapter 7.

Regardless of the method of cruise control, the range is calculated by solving Eq 4.115.

$$\text{Range} = \int_{W_1}^{W_2} (\text{S.R. } \delta) \left(\frac{W}{\delta}\right) \frac{1}{W} \, dW \tag{Eq 4.115}$$

Since range factor was defined as:

$$\text{R.F.} = \left[(\text{S.R.}\ \delta) \frac{W}{\delta} \right] \tag{Eq 4.118}$$

and Eq 4.118 resulted in an analytical solution to the range equation for a schedule (1) cruise for constant R.F.:

$$\text{Range} = \text{R.F.} \ln \frac{W_1}{W_2} \tag{Eq 4.126}$$

Non optimum solutions can be obtained from a graphical solution to Eq 4.114.

$$\text{Range} = \int_{W_1}^{W2} (\text{S.R.})\, dW \tag{Eq 4.114}$$

Where:

δ	Pressure ratio	
R.F.	Range factor	
S.R.	Specific range	nmi/lb
W	Weight	lb.

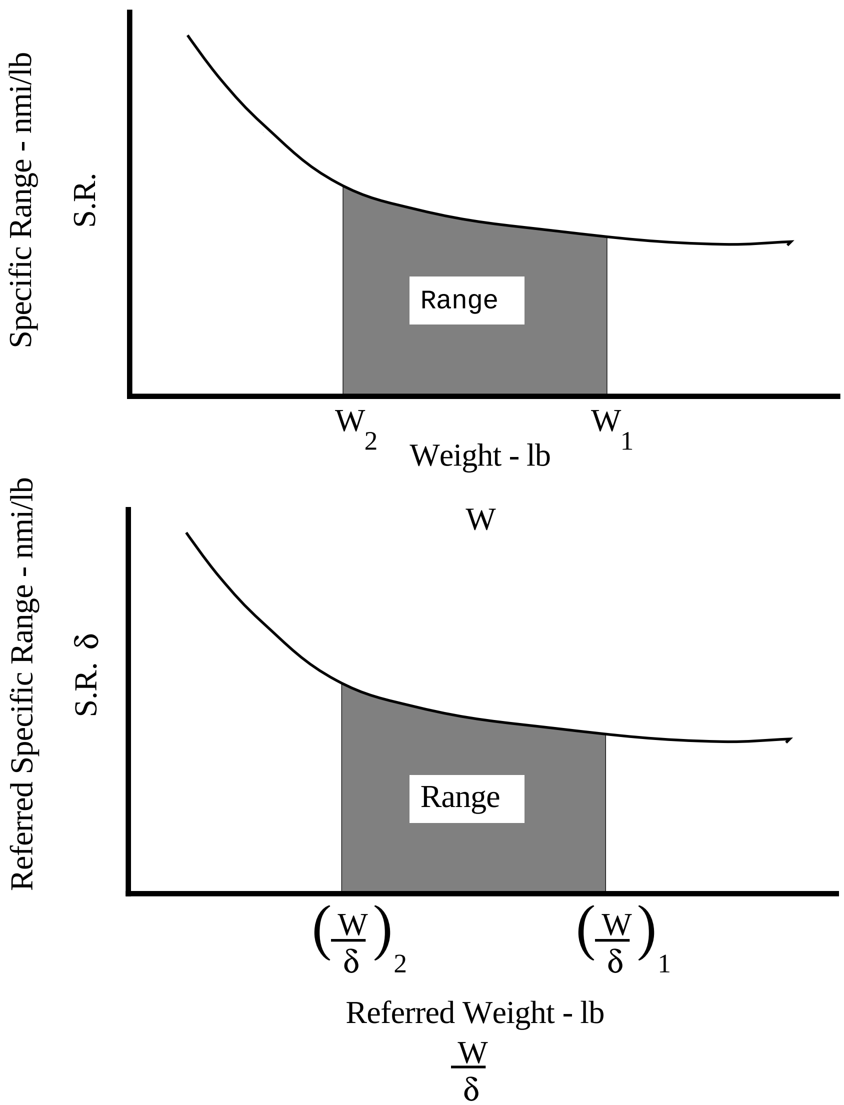

Figure 4.60
ACTUAL RANGE FROM NON OPTIMUM CRUISE

The shaded area in each of the curves of figure 4.60 represent the same thing. Both show the actual range. The problem can be solved from the referred data. Using schedule (2) or (3), the relationship of ((S.R.(δ)) and (W/δ) can be plotted as the fuel is burned (decreasing W/δ). The results would appear as in figure 4.61 and can be graphically integrated to get the range for various cruise schedules. Non-optimum cruise and the cruise climb can be compared to determine if the aircraft under test has any realistic benefit from the cruise climb.

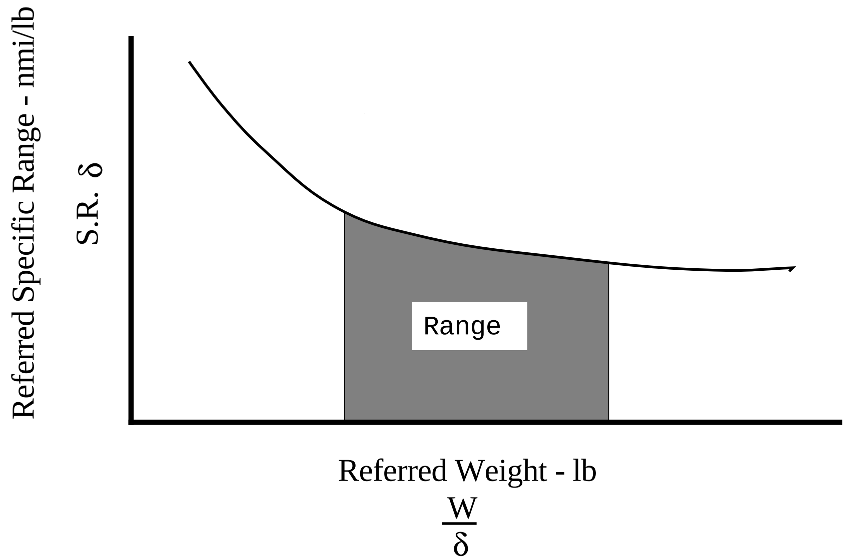

Figure 4.61
RANGE FOR VARIOUS CRUISE SCHEDULES

4.6.8 TURBOPROP RANGE AND ENDURANCE

Unreferring the data presented in section 4.5.4 allows the calculation of actual range and endurance for specified conditions. Input GW, altitude of interest and ambient temperature to the computer program. Curves of SHP, fuel flow, and specific range as functions of calibrated airspeed (V_c) can be produced. Combined curves for a series of set conditions define the range and endurance over the entire aircraft operating envelope. Typical unreferred curves are shown in figures 4.62, 4.63, and 4.64.

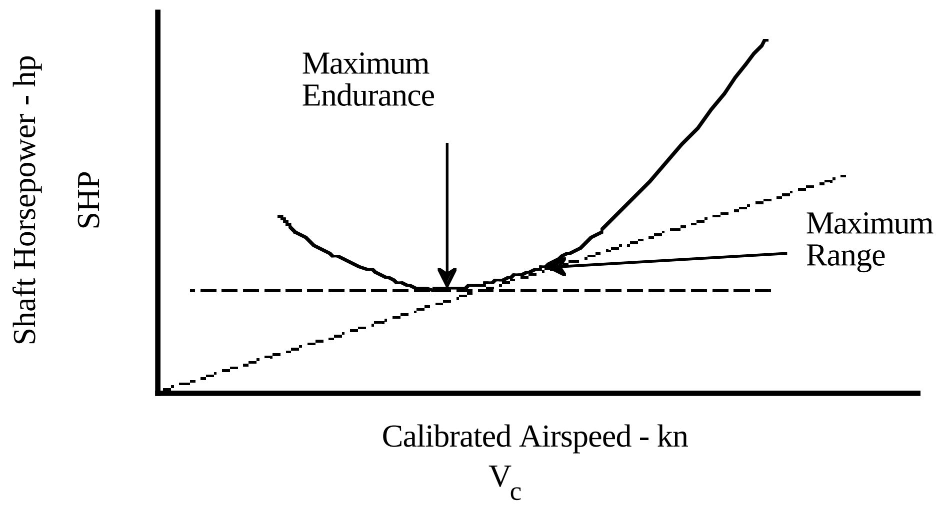

Figure 4.62
POWER REQUIRED

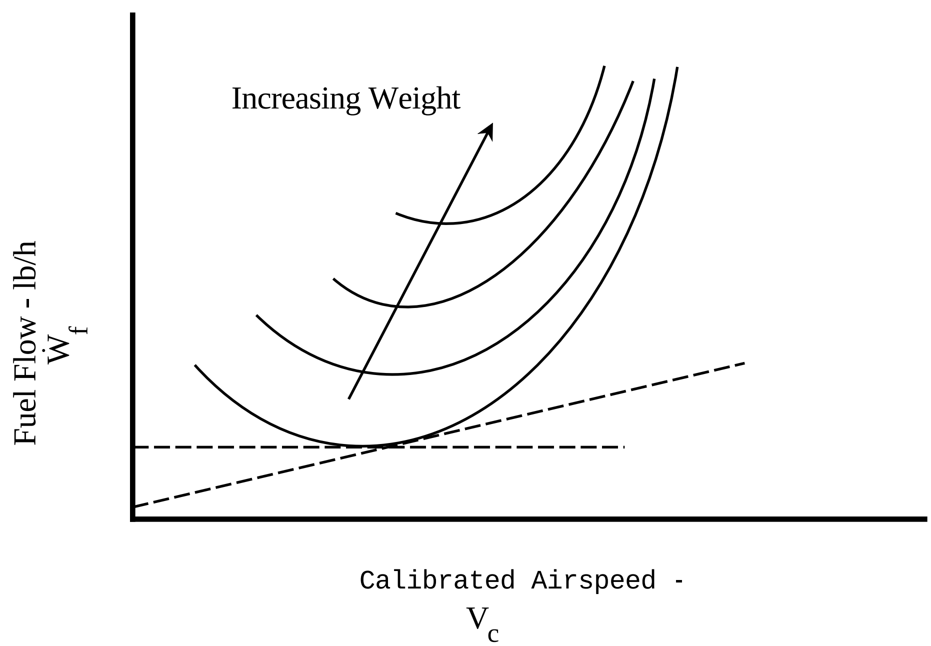

Figure 4.63
FUEL CONSUMPTION

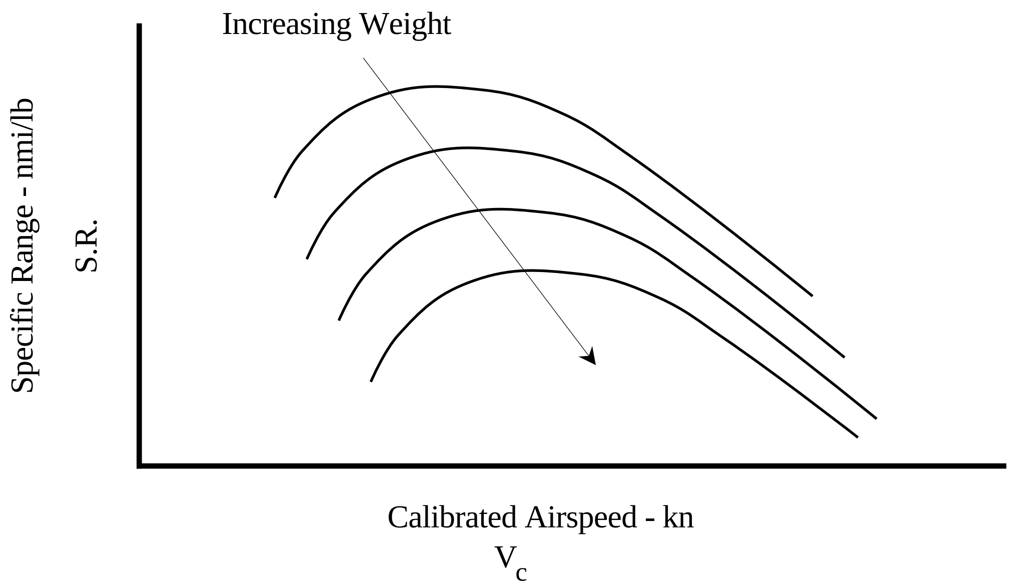

Figure 4.64
SPECIFIC RANGE

From these curves, range and endurance profiles can be determined as a function of fuel used for representative mission altitudes and ambient conditions expected.

4.6.8.1 MAXIMUM VALUES

Maximum values can be determined for a given set of conditions from the unreferred plots as follows:

1.	Maximum endurance	Specified at given ˚C, ΔT_a
		Fuel flow in lb/h
		Velocity for maximum endurance
		Maximum endurance in hours
2.	Maximum range	Specific range in nmi/lb
		Velocity for maximum range
		Maximum range in nmi

4.6.8.2 WIND EFFECTS

The effect of wind on range of the turboprop aircraft may be even more important than for the turbojet due to the lower airspeed. A head wind reduces range and a tail wind increases range. Specific range for no wind was $\frac{V_T}{\dot{W}_f}$. For a head wind or tail wind ground speed for the fuel flow needs to be maximized $\frac{GS}{\dot{W}_f}$. The variation with speed to maximize specific range in the presence of wind is similar to that shown for the turbojet in figure 4.48.

4.6.8.3 PROPELLER EFFICIENCY

The turbine of a turboprop is designed to absorb large amounts of energy from the expanding combustion gases in order to provide not only the power required to satisfy the compressor and other components of the engine, but to deliver the maximum torque possible to a propeller shaft as well. Propulsion is produced through the combined action of a propeller at the front and the thrust produced by the unbalanced forces created within the engine that result in the discharge of high-velocity gases through a nozzle at the rear. The propeller of a typical turboprop is responsible for roughly 90% of the total thrust under sea level, static conditions on a standard day. This percentage varies with airspeed, exhaust nozzle area and, to some degree temperature, barometric pressure and the power rating of the engine.

Section 4.6.7 assumes constant propeller efficiency. Test results would need to be spot checked at various conditions of weight and altitude to confirm results. If follow-on results show significant differences in propeller efficiency, a computer iteration with the additional results could determine the actual performance. In this case a combination test may have to be performed. W/δ type tests would be appropriate.

4.7 MISSION SUITABILITY

The mission requirements are the ultimate standard for level flight performance. Obviously, the requirements of a trainer, fighter or attack aircraft vary. Optimum range and endurance profiles are not feasible for every mission phase. However, the level flight performance must satisfy mission requirements. Often, aircraft roles change as they mature or the threat is changed. With this in mind, the level flight performance capability would ideally be greater than the procuring agency originally set as the goals. Level flight performance is only a part of the overall performance and the suitability of the aircraft for a particular role also depends on takeoff, excess energy, maneuvering, climb, descent, and landing performance.

The specifications set desired numbers the procuring agency is expecting of the final product. These specifications are important as measures for contract performance. They are important in determining whether or not to continue the acquisition process at various stages of aircraft development.

Mission suitability conclusions concerning level flight performance are not restricted to optimum performance test results and specification conformance. Test results reflect the performance capabilities of the aircraft, while mission suitability conclusions include the flying qualities associated with specific airspeeds and shipboard operations. Consideration of the following items is worthwhile when recommending airspeeds for maximum endurance and range:

1. Flight path stability.
2. Pitch attitude.
3. Field of view.
4. IFR/VFR holding.
5. Mission profile or requirements.
6. Overall performance including level flight.
7. Compatibility of airspeeds / altitudes with the mission and location restrictions.
8. Performance sensitivities for altitude or airspeed deviations.

4.8 SPECIFICATION COMPLIANCE

Level flight performance guarantees are stated in the detailed specification for the model and in Naval Air Systems Command Specification, AS-5263. The detail specification provides mission profiles to be expected and performance guarantees generically as follows:

1. Mission requirements.
    a. Land or sea based.
    b. Ferry capable.
    c. Instrument departure, transit, and recovery.
    d. Type of air combat maneuvering.
    e. Air to air combat (offensive and/or defensive) including weapons deployment.
    f. Low level navigation.
    g. Carrier suitability.

2. Performance guarantees are based on: type of day (ICAO standard atmosphere), empty gross weight, standard gross weight, drag index, fuel quantity and type at engine start, engine(s) type, loading, and configuration. Guarantees would likely include:
    a. Maximum Mach number specified in level flight at a specified gross weight and power setting at prescribed altitude.
    b. Maximum range at optimum cruise altitude, at specified Mach. Range specified as not less than, given the fuel allowed for start, taxi, maximum or military thrust take off to climb speed, power to be used in climb to optimum altitude, and fuel reserve requirements.
    c. Maximum specific range in nautical miles per pound of fuel at a prescribed altitude at standard gross weight.
    d. Maximum endurance fuel flow in pounds per hour at prescribed gross weight and airspeeds at given altitudes.

AS-5263 further defines requirements for, and methods of, presenting characteristics and performance data for U.S. Navy piloted aircraft. Deviations from this specification are permissible, but in all cases must be approved by the procuring activity. Generally level flight performance guarantees are stated for standard day conditions and mission or takeoff gross weight. The specification gives some general guidelines to performance guarantees. It states the following about level flight:

1. All speeds are in knots true airspeed unless otherwise noted.
2. Maximum speed is the highest speed obtainable in level flight. The maximum speed shall be within all operating restrictions.

3. Combat speed is the highest speed obtainable in level flight at combat weight with maximum power at combat altitude.

4. Average cruise speed is determined by dividing the total cruise distance by the time for cruise not including time and distance to climb.

5. Speed at specified altitude is the highest speed obtainable within all operating restrictions in level flight at combat weight and stated power. Except for interceptors, the altitude at which this speed is quoted shall not exceed either combat or cruise ceilings. For interceptors the altitude shall not exceed combat ceiling.

6. Airspeed for long range operation is the greater of the two speeds at which 99 percent of the maximum miles per pound of fuel are attainable at the momentary weight and altitude.

7. Airspeed for maximum endurance operation is the airspeed for minimum fuel flow attainable at momentary weight and altitude except as limited by acceptable handling characteristics of the aircraft.

8. Fuel consumption data shall be increased by 5% for all engine power conditions as a service tolerance to allow for practicable operation. Additionally, corrections or allowances shall be made for power plant installation losses such as accessory drives, ducts, fans, cabin pressure bleeds, tail-pipes, afterburners, ram-pressure recovery, etc.

9. Combat radius is the distance attainable on a practicable flight to the target and return a distance equal to that flown out carrying a specific load.

10. Combat range is the distance (including climb distance) attainable on a practicable one way flight carrying a specific load.

4.9 GLOSSARY

4.9.1 NOTATIONS

550	Conversion factor	$550 \frac{\text{ft-lb}}{\text{s}} = 1$ horsepower
AR	Aspect ratio	
a_{ssl}	Standard sea level speed of sound	661.483 kn
BHP	Brake horsepower	hp
C_D	Drag coefficient	
C_{D_i}	Induced drag coefficient	

C_{D_M}	Mach drag coefficient	
C_{D_p}	Parasite drag coefficient	
$C_{D_{p\,(M)}}$	Parasite drag coefficient at high Mach	
C_L	Lift coefficient	
D	Drag	lb
$\Delta H_{P_{ic}}$	Altimeter instrument correction	ft
ΔH_{pos}	Altimeter position error	ft
D_i	Induced drag	lb
D_M	Mach drag	lb
D_p	Parasite drag	lb
ΔT_a	Temperature differential	
ΔT_{ic}	Temperature instrument correction	˚C
Δt_j	Time of each time interval	s
ΔV_{ic}	Airspeed instrument correction	kn
ΔV_{pos}	Airspeed position error	kn
e	Oswald’s efficiency factor	
$e_{(M)}$	Oswald’s efficiency factor at high Mach	
EGT	Exhaust gas temperature	˚C
EPR	Engine pressure ratio	
F/C	Fuel counter	lb
GS	Ground speed	kn
H_P	Pressure altitude	ft
H_{P_c}	Calibrated pressure altitude	ft
H_{P_i}	Indicated altitude	ft
H_{P_o}	Observed pressure altitude	ft
ICAO	International Civil Aeronautics Organization	
J	Propeller advance ratio	
K	Constant	
K_1	Parasite drag constant	
K_2	Induced drag constant	
K_3	Constant	
K_4	Constant	
K_o	Constant	
K_T	Temperature recovery factor	
L	Lift	lb

M	Mach number	
M_{max}	Mach number at maximum thrust	
M_{mrt}	Mach number at military rated thrust	
M_o	Observed Mach number	
MSL	Mean sea level	
N	Engine speed	RPM
n	Number of time intervals	
nmi	Nautical miles	
$\frac{N}{\sqrt{\theta}}$	Referred engine speed	RPM
OAT	Outside air temperature	˚C or ˚K
P	Pressure	psf
P_a	Ambient pressure	psf
P_{ssl}	Standard sea level pressure	2116.217 psf
P_T	Total pressure	psf
q	Dynamic pressure	psf
R.F.	Range factor	
$R.F._{Test}$	Test day average range factor	
R_e	Reynold's number	
R_{Std}	Standard day cruise range	nmi
R_T	Total range	nmi
R_{Test}	Test cruise range	nmi
S	Wing area	ft^2
S.E.	Specific endurance	h/lb
S.R.	Specific range	nmi/lb
SHP	Shaft horsepower	hp
SHP_e	Equivalent shaft horsepower	hp
SHPSFC	Shaft horsepower specific fuel consumption	$\frac{lb/h}{hp}$
T	Thrust	lb
t	Time	s
T_a	Ambient temperature	˚C or ˚K
$T_{a_{Std}}$	Standard ambient temperature	˚K
T_G	Gross thrust	lb
THP	Thrust horsepower	hp
THP_e	Equivalent thrust horsepower	hp

THP_i	Induced thrust horsepower	hp
THP_{min}	Minimum thrust horsepower	hp
THP_p	Parasite thrust horsepower	hp
THPSFC	Thrust horsepower specific fuel consumption	$\frac{lb/h}{hp}$
Time	Time at the start of each segment	s
T_{N_x}	Net thrust parallel flight path	lb
$\frac{T_{N_x}}{\delta}$	Referred net thrust parallel flight path	lb
T_o	Observed temperature	˚C
T_R	Ram drag	lb
TSFC	Thrust specific fuel consumption	$\frac{lb/h}{lb}$
T_{ssl}	Standard seal level temperature	15˚C, 288.15˚K
T_T	Total temperature	˚K
t_T	Total cruise time	s
T_{T_2}	Inlet total temperature (at engine compressor face)	˚K
V	Velocity	kn
V_c	Calibrated airspeed	kn
V_e	Equivalent airspeed	kn
V_H	Maximum level flight airspeed	kn
V_i	Indicated airspeed	kn
Vj	Avg true airspeed in time interval	kn
V_{max}	Maximum airspeed	kn
V_{min}	Minimum airspeed	kn
V_o	Observed airspeed	kn
V_T	True airspeed	kn
V_v	Vertical velocity	fpm
W	Weight	lb
W/δ	Weight to pressure ratio	lb
W_1	Initial cruise weight	lb
W_2	Final cruise weight	lb
$W_{aircraft}$	Aircraft weight	lb
W_f	Fuel weight	lb
$W_{f_{Used}}$	Fuel used	lb

$\dot{W}_f$	Fuel flow	lb/h
$\dot{W}_{f\ ref}$	Referred fuel flow	lb/h
W_{ref}	Referred aircraft weight	lb
W_{Std_1}	Standard initial cruise weight	lb
W_{Std_2}	Standard final cruise weight	lb
¢	Constant	

4.9.2 GREEK SYMBOLS

α (alpha)	Angle of attack	deg
α_i	Induced angle of attack	deg
α_j	Thrust angle	deg
δ (delta)	Pressure ratio	
γ (gamma)	Ratio of specific heats	
η_P (eta)	Propeller efficiency	
μ (mu)	Viscosity	lb-s/ft^2
π (pi)	Constant	
θ (theta)	Temperature ratio	
θ_T	Total temperature ratio	
ρ (rho)	Air density	slugs/ft^3
ρ_a	Ambient air density	slugs/ft^3
ρ_{ssl}	Standard sea level air density	0.0023769 slugs/ft^3
σ (sigma)	Density ratio	

4.10 REFERENCES

1. Department of the Army, *Operator's Manual for Army U-21A Aircraft*, TM 555-1510-209-10, Headquarters Department of the Army, Washington D.C., 26 September 1988.

2. Naval Air System Command Specification, *Guidelines For Preparation Of Standard Aircraft Characteristics Charts And Performance Data Piloted Aircraft (Fixed Wing), AS-5263*, 23 October, 1986.

3. Naval Test Pilot School Flight Test Manual, *Fixed Wing Performance, Theory and Flight Test Techniques*, USNTPS-FTM-No.104, U.S. Naval Test Pilot School, Patuxent River, MD, July 1977.

4. Powell, J.W., *Airplane Performance, Fixed Wing Flight Mechanics*, USNTPS Class Notes, USNTPS, Patuxent River, MD., no date.

5. Roberts, S.C., *Light Aircraft Performance for Test Pilots and Flight Test Engineers*, Flight Research, Inc, Mojave, CA., 9 July 1982.

6. USAF Test Pilot School Flight Test Manual, *Performance Flight Testing Phase*, Volume I, Chapter 11, USAF Test Pilot School, Edwards AFB, CA, February 1987.

7. USAF Test Pilot School, *Performance Phase Textbook Volume I*, USAF TPS-CUR-86-01, USAF, Edwards AFB, CA, April, 1986.

8. USAF Test Pilot School, *Performance Planning Guide*, USAF Test Pilot School, Edwards AFB, CA.,July 1987.