CHAPTER 5

EXCESS POWER CHARACTERISTICS

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FIGURES

CHAPTER 5

EQUATIONS

$\sin\gamma = \dfrac{V_T\,(\text{vertical})}{V_T\,(\text{flight path})} = \dfrac{\frac{dh}{dt}}{V_T}$	(Eq 5.13)	5.9
$T_{N_x} - D - W\dfrac{dh}{dt}\dfrac{1}{V_T} = \dfrac{W}{g}\dfrac{dV_T}{dt}$	(Eq 5.14)	5.9
$\dfrac{V_T\left(T_{N_x} - D\right)}{W} - \dfrac{dh}{dt} = \dfrac{V_T}{g}\dfrac{dV_T}{dt}$	(Eq 5.15)	5.9
$\dfrac{V_T\left(T_{N_x} - D\right)}{W} = \dfrac{dE_h}{dt}$	(Eq 5.16)	5.9
$P_s = \dfrac{V_T\left(T_{N_x} - D\right)}{W}$	(Eq 5.17)	5.9
$P_s = \dfrac{dE_h}{dt}$	(Eq 5.18)	5.10
$P_s = \dfrac{dh}{dt} + \dfrac{V_T}{g}\dfrac{dV_T}{dt}$	(Eq 5.19)	5.10
$P_s = \dfrac{V_T\left(T_{N_x} - D - \Delta D_i\right)}{W}$	(Eq 5.20)	5.13
$P_s = \dfrac{V_T\left(T_{N_x} - D - \Delta D_i\right)}{W + \Delta W}$	(Eq 5.21)	5.15
$P_s = \dfrac{V_T\left(T_{N_x} - D - \Delta D_p\right)}{W}$	(Eq 5.22)	5.16
$P_s = \dfrac{V_T\,(T_{N_x} + \Delta T_{N_x} - D)}{W}$	(Eq 5.23)	5.17
$P_s = \dfrac{\left(P_A - \Delta P_A\right) - \left(P_{req} + \Delta P_{req}\right)}{W}$	(Eq 5.24)	5.19

$P_{s_{Std}} = P_{s_{Test}} \frac{W_{Test}}{W_{Std}} \sqrt{\frac{T_{a_{Std}}}{T_{a_{Test}}}} + \frac{V_{T_{Std}}}{W_{Std}} (\Delta T_{N_x} - \Delta D)$	(Eq 5.36)	5.32
$V_c = V_i + \Delta V_{pos}$	(Eq 5.37)	5.36
$H_{P_c} = H_{P_i} + \Delta H_{pos}$	(Eq 5.38)	5.36
$M = f\left(V_c , H_{P_c}\right)$	(Eq 5.39)	5.37
$W_{Test} = \text{Initial } W - \int \dot{W}_f \, dt$	(Eq 5.40)	5.37
$°C = °K - 273.15$	(Eq 5.41)	5.37
$OAT = f\left(T_a , M_T\right)$	(Eq 5.42)	5.37
$T_a = f\left(OAT, M \right)$	(Eq 5.43)	5.37
$V_{T_{Test}} = f\left(V_c , H_{P_c} , T_a\right)$	(Eq 5.44)	5.37
$V_{T_{Std}} = f\left(V_c, H_{p_c}, T_{Std}\right)$	(Eq 5.45)	5.37
$h = H_{P_{c_{ref}}} + \Delta H_{P_c} \left(\frac{T_a}{T_{Std}}\right)$	(Eq 5.46)	5.37
$E_h = h + \frac{V_{T_{Test}}^2}{2g}$	(Eq 5.47)	5.38
$P_{s_{Test}} = \frac{dE_h}{dt}$	(Eq 5.48)	5.38
$\gamma_{Test} = \sin^{-1} \left(\frac{dh/dt}{V_{T_{Test}}}\right)$	(Eq 5.49)	5.38

CHAPTER 5

EXCESS POWER CHARACTERISTICS

5.1 INTRODUCTION

This chapter deals with determining excess power characteristics using the total energy approach. Test techniques commonly used are presented with the associated methods of data reduction and analysis.

5.2 PURPOSE OF TEST

The purpose of these tests is to determine the aircraft excess power characteristics, with the following objectives:

1. Derive climb schedules to optimize time to height, energy gain, or to minimize fuel consumption (Chapter 7).
2. Predict sustained turn performance envelopes (Chapter 6).
3. Define mission suitability and enable operational comparisons to be made among different aircraft.

5.3 THEORY

Formerly, climb performance and level acceleration were treated as separate and distinct performance parameters, each with their own unique flight test requirements, data reduction, and analysis. In the 1950's it was realized climb performance and level acceleration were really different aspects of the same characteristic, and aircraft performance is based on “the balance that must exist between the kinetic and potential energy exchange of the aircraft, the energy dissipated against the drag, and the energy derived from the fuel” (Reference 1, pp 187-195). Excess power characteristics are tested and analyzed efficiently using the total energy concept rather than treating climbs and accelerations as separate entities.

5.3.1 TOTAL ENERGY

The total energy possessed by an aircraft inflight is the sum of its potential energy (energy of position) and its kinetic energy (energy of motion) and is expressed as:

$$TE = PE + KE \tag{Eq 5.1}$$

Where:

KE	Kinetic energy	ft-lb
PE	Potential energy	ft-lb
TE	Total energy	ft-lb.

5.3.1.1 POTENTIAL ENERGY

Potential energy (PE) is the energy a body possesses by virtue of its displacement against a field from a reference energy level. The reference energy level is usually the lowest available; although, sometimes only a local minimum. An aircraft inflight is displaced against the earth’s gravitational field from a reference level, which is usually chosen as sea level. The aircraft's potential energy is equal to the work done in raising it to the displaced level, expressed as:

$$PE = \int_{o}^{h} W\,dh \tag{Eq 5.2}$$

In terms of measurable flight test quantities, Eq 5.2 can be expressed as:

$$PE = W\left(H_{P_c} + \Delta T \text{ correction}\right) \tag{Eq 5.3}$$

Where:

h	Tapeline altitude	ft
H_{P_c}	Calibrated pressure altitude	ft
PE	Potential energy	ft-lb
T	Temperature	˚C or ˚K
W	Weight	lb.

5.3.1.2 KINETIC ENERGY

The kinetic energy (KE) or translational energy along the flight path is expressed as:

$$KE = \frac{1}{2}\frac{W}{g}V_T^2 \qquad \text{(Eq 5.4)}$$

Where:

g	Gravitational acceleration	ft/s^2
KE	Kinetic energy	ft-lb
V_T	True airspeed	ft/s
W	Weight	lb.

5.3.2 SPECIFIC ENERGY

Using Eq 5.3 and 5.4, total energy (TE) can be expressed as:

$$TE = W\,h + \frac{1}{2}\frac{W}{g}V_T^2 \qquad \text{(Eq 5.5)}$$

Using standard American engineering units (velocity in ft/s, weight in lb, acceleration in ft/s^2), energy has units of ft-lb. To generalize the analysis and eliminate dependence on aircraft weight, Eq 5.5 is normalized by dividing by weight:

$$\frac{TE}{W} = h + \frac{V_T^2}{2\,g} \qquad \text{(Eq 5.6)}$$

Eq 5.6 is the specific energy state equation, and has units of feet. The aircraft's specific energy, or energy per unit weight, can be defined in terms of energy height (E_h):

$$E_h = h + \frac{V_T^2}{2\,g} \qquad \text{(Eq 5.7)}$$

Where:

E_h	Energy height	ft
g	Gravitational acceleration	ft/s^2
h	Tapeline altitude	ft
TE	Total energy	ft-lb
V_T	True airspeed	ft/s
W	Weight	lb.

Energy height is not an altitude, rather the sum of the aircraft's specific potential and kinetic energies. It represents the altitude which the aircraft theoretically would be capable of reaching in a zoom climb, if its kinetic energy were perfectly convertible to potential energy without loss of any kind, and if it arrived at that altitude at zero airspeed.

Energy paper, consisting of lines of constant E_h superposed on a height-velocity plot, is used commonly for analysis. The units on the horizontal axis may be either true airspeed or Mach number. The shape of the lines of constant E_h are parabolic for a true airspeed plot but not for a Mach number plot, because of the temperature dependent relationship between Mach number and true airspeed below the tropopause. Some examples of energy paper are shown in figures 5.1 and 5.2.

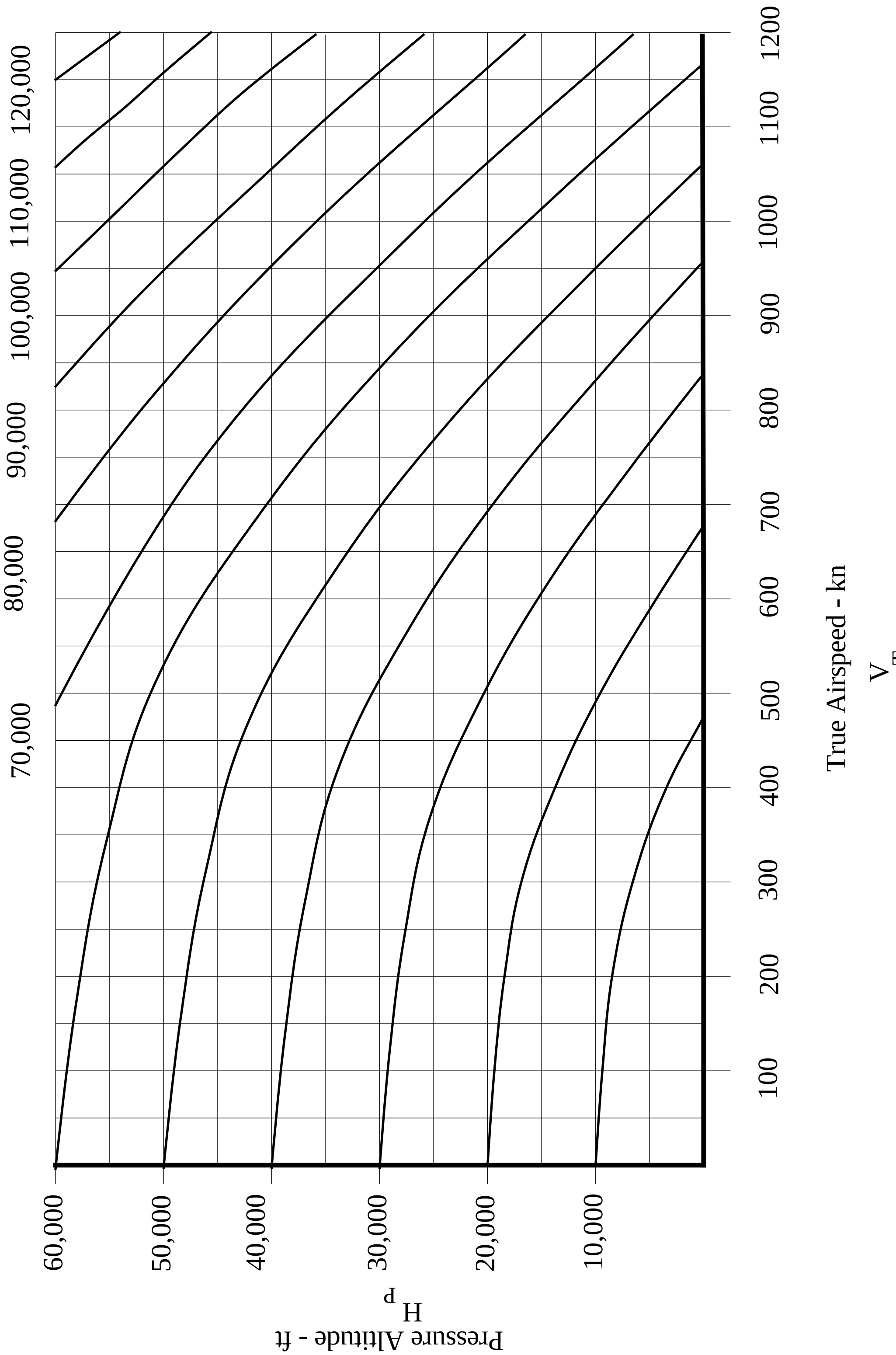

Figure 5.1

ENERGY HEIGHT VERSUS TRUE AIRSPEED

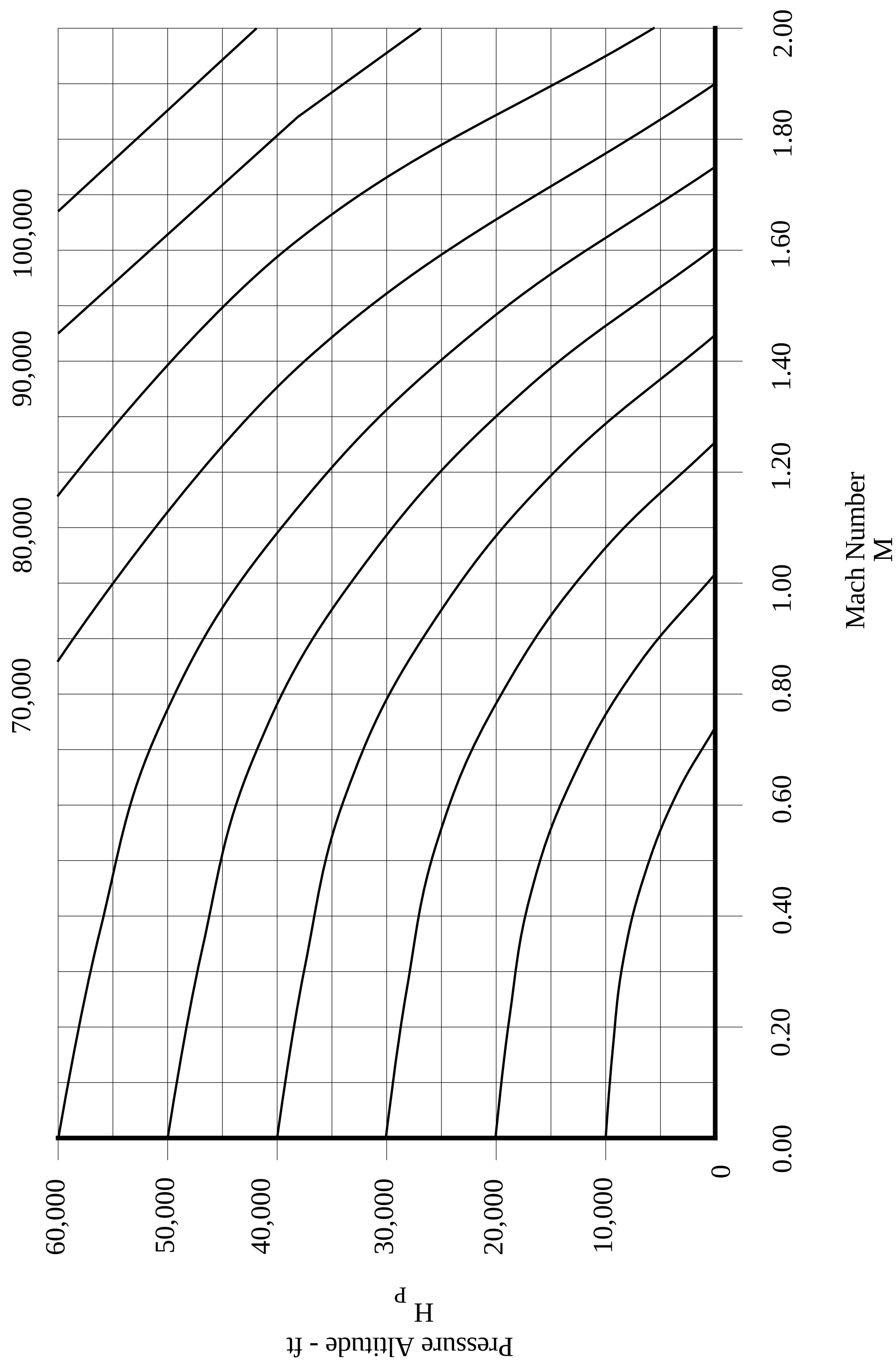

Figure 5.2

ENERGY HEIGHT VERSUS MACH NUMBER

5.3.3 SPECIFIC POWER

Power is defined as the rate of doing work, or the time rate of energy change. The specific power of an aircraft is obtained by taking the time derivative of the specific energy equation, Eq 5.7, which becomes:

$$\frac{d}{dt}E_h = \frac{d}{dt}\left(h + \frac{V_T^2}{2\,g}\right) \qquad \text{(Eq 5.8)}$$

Or:

$$\frac{d}{dt}E_h = \frac{dh}{dt} + \frac{V_T}{g}\frac{dV_T}{dt} \qquad \text{(Eq 5.9)}$$

Where:

E_h	Energy height	ft
g	Gravitational acceleration	ft/s^2
h	Tapeline altitude	ft
t	Time	s
V_T	True airspeed	ft/s.

The units are now ft/s, which does not denote a velocity but rather the specific energy rate in $\frac{\text{ft-lb}}{\text{lb-s}}$. Eq 5.9 contains terms for both rate of climb and flight path acceleration.

5.3.4 DERIVATION OF SPECIFIC EXCESS POWER

Consider an aircraft accelerating in a climbing left turn as shown in figure 5.3.

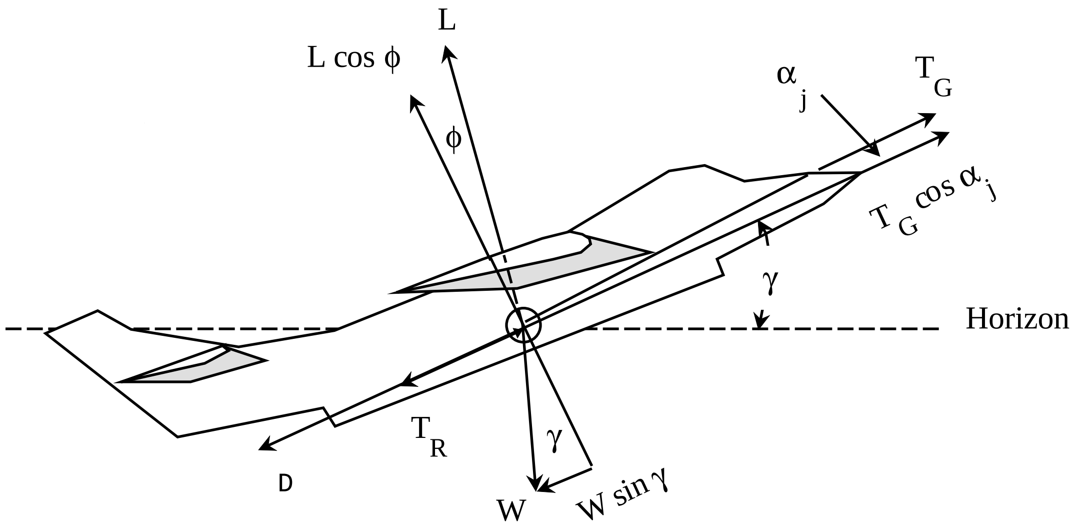

Figure 5.3
AIRCRAFT ACCELERATING IN CLIMBING LEFT TURN

Assuming constant mass ($\frac{V}{g}\frac{dW}{dt} = 0$), the forces parallel to flight path (F_x) are resolved as:

$$\sum F_x = \frac{W}{g}\frac{dV_T}{dt} \qquad \text{(Eq 5.10)}$$

Expanding the left hand side and noting the lift forces, L and L cos ϕ, act perpendicular to the flight path there is no component along the flight path:

$$T_{N_x} = T_G \cos \alpha_j - T_R \qquad \text{(Eq 5.11)}$$

$$T_{N_x} - D - W \sin \gamma = \frac{W}{g}\frac{dV_T}{dt} \qquad \text{(Eq 5.12)}$$

The flight path angle (γ) can be expressed in terms of the true vertical and true flight path velocities:

$$\sin \gamma = \frac{V_T \text{ (vertical)}}{V_T \text{ (flight path)}} = \frac{\frac{dh}{dt}}{V_T} \tag{Eq 5.13}$$

Assuming the angle between the thrust vector and the flight path (α_j) is small (a good assumption for non-vectored thrust), then $\cos \alpha_j \approx 1$.

Substituting these results in Eq 5.12 yields:

$$T_{N_x} - D - W \frac{dh}{dt} \frac{1}{V_T} = \frac{W}{g} \frac{dV_T}{dt} \tag{Eq 5.14}$$

Eq 5.14, normalized by dividing throughout by the aircraft weight, multiplied throughout by the true airspeed, and rearranged produces:

$$\frac{V_T \left(T_{N_x} - D \right)}{W} - \frac{dh}{dt} = \frac{V_T}{g} \frac{dV_T}{dt} \tag{Eq 5.15}$$

From Eq 5.9:

$$\frac{V_T \left(T_{N_x} - D \right)}{W} = \frac{dE_h}{dt} \tag{Eq 5.16}$$

The left hand side of Eq 5.16 represents the net force along the flight path (excess thrust, which may be positive or negative), which when multiplied by the velocity yields the excess power, and divided by the aircraft weight becomes the specific excess power of the aircraft (P_s):

$$P_s = \frac{V_T \left(T_{N_x} - D \right)}{W} \tag{Eq 5.17}$$

Or:

$$P_s = \frac{dE_h}{dt} \qquad \text{(Eq 5.18)}$$

Which can be expressed as:

$$P_s = \frac{dh}{dt} + \frac{V_T}{g}\frac{dV_T}{dt} \qquad \text{(Eq 5.19)}$$

Where:

α_j	Thrust angle	
D	Drag	lb
E_h	Energy height	ft
F_x	Forces parallel to flight path	lb
γ	Flight path angle	deg
g	Gravitational acceleration	ft/s^2
h	Tapeline altitude	ft
P_s	Specific excess power	ft/s
t	Time	s
T_G	Gross thrust	lb
T_{N_x}	Net thrust parallel flight path	lb
T_R	Ram drag	lb
V_T	True airspeed	ft/s
W	Weight	lb.

The terms in Eq 5.19 all represent instantaneous quantities. P_s relates how quickly the airplane can change its energy state. P_s is a measure of what is known as energy maneuverability. When $P_s > 0$, the airplane is gaining energy. When $P_s < 0$, the airplane is losing energy. A typical P_s plot is shown in figure 5.4.

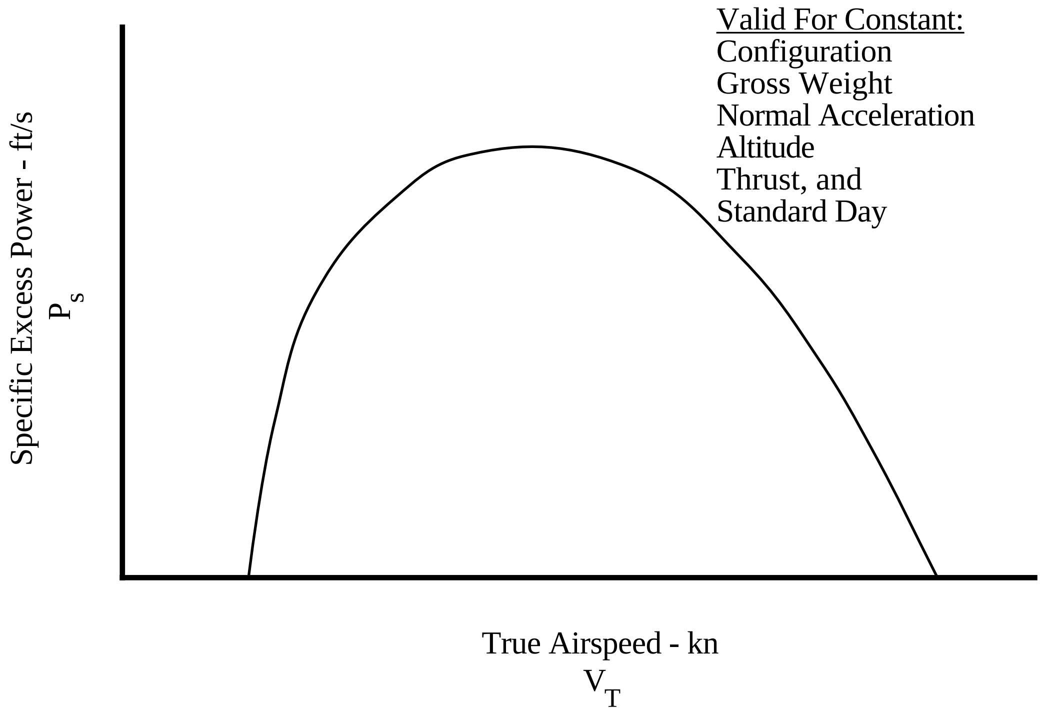

Figure 5.4
SPECIFIC EXCESS POWER VERSUS TRUE AIRSPEED

P_S is valid for a single flight condition (configuration, gross weight, normal acceleration, and altitude). A family of P_S plots at altitude intervals of approximately 5,000 ft is necessary to define the airplane's specific excess power envelope for each configuration. When presented as a family, P_S curves usually are plotted versus Mach number or calibrated airspeed (V_C) as shown in figure 5.5.

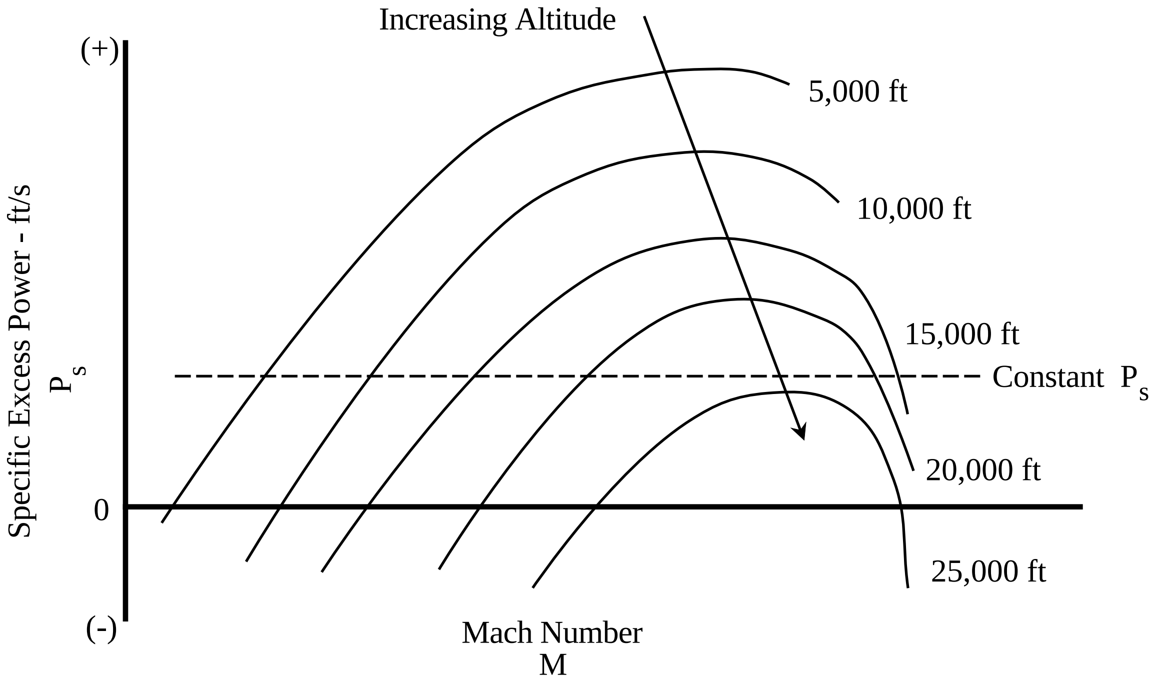

Figure 5.5

FAMILY OF SPECIFIC EXCESS POWER CURVES

Notice the P_S curves have a similar shape, but shift and decrease in magnitude with increasing altitude. Regions can be documented where the airplane is instantaneously losing energy, represented in figure 5.5 by the conditions where P_S is negative. The variation of P_S with Mach number and altitude are often displayed on a plot of energy height versus Mach number for climb performance analysis. For a given level of P_S, represented by a horizontal cut in figure 5.5, combinations of altitude and Mach number can be extracted to show the change in energy height. A plot containing several such P_S contours is used to determine climb profiles (Chapter 7). P_S is derived analytically from the airplane thrust available and thrust required curves (Reference 2, Chapter 10) which are multiplied by the velocity to obtain the power available and power required curves. The difference between power available and power required, divided by the aircraft weight, is the specific excess power, P_S. The graphical portrayal of typical P_S curves for jet and propeller aircraft is presented in figure 5.6.

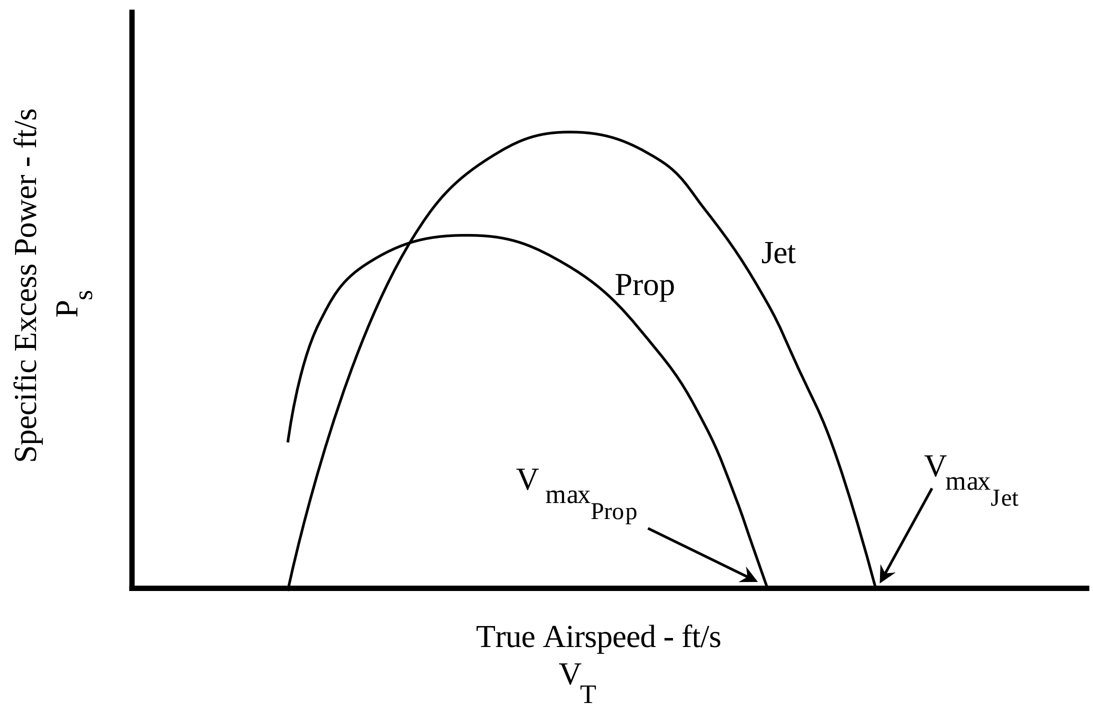

Figure 5.6
TYPICAL SPECIFIC EXCESS POWER CHARACTERISTICS

5.3.5 EFFECTS OF PARAMETER VARIATION ON SPECIFIC EXCESS POWER

The following discussion of the effect of variation in normal acceleration, gross weight, drag, thrust, and altitude is presented as it applies to jet airplanes.

5.3.5.1 INCREASED NORMAL ACCELERATION

Increased normal acceleration affects the P_S equation by increasing the induced drag and has most effect at low speeds (Figure 5.7):

$$P_s = \frac{V_T\left(T_{N_x} - D - \Delta D_i\right)}{W} \qquad \text{(Eq 5.20)}$$

Where:

D	Drag	lb
ΔD_i	Change in induced drag	lb
P_s	Specific excess power	ft/s
T_{N_x}	Net thrust parallel flight path	lb
V_T	True airspeed	ft/s
W	Weight	lb.

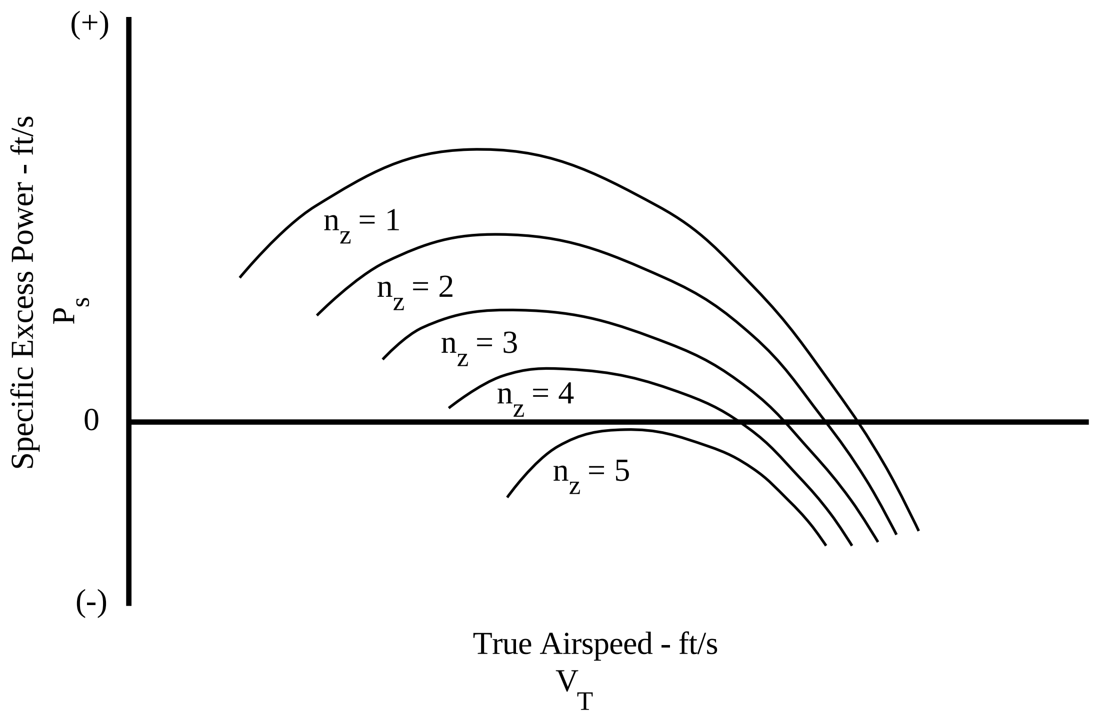

Figure 5.7
EFFECT OF INCREASED NORMAL ACCELERATION ON SPECIFIC EXCESS POWER

Chapter 6 contains a discussion of P_s for $n_z > 1$.

5.3.5.2 INCREASED GROSS WEIGHT

The effect of increasing gross weight is similar to that of increasing the normal acceleration, with the difference that both the numerator and denominator are affected rather than the numerator alone (Figure 5.8):

$$P_s = \frac{V_T\left(T_{N_x} - D - \Delta D_i\right)}{W + \Delta W} \qquad \text{(Eq 5.21)}$$

Where:

D	Drag	lb
ΔD_i	Change in induced drag	lb
P_s	Specific excess power	ft/s
T_{N_x}	Net thrust parallel flight path	lb
V_T	True airspeed	ft/s
W	Weight	lb.

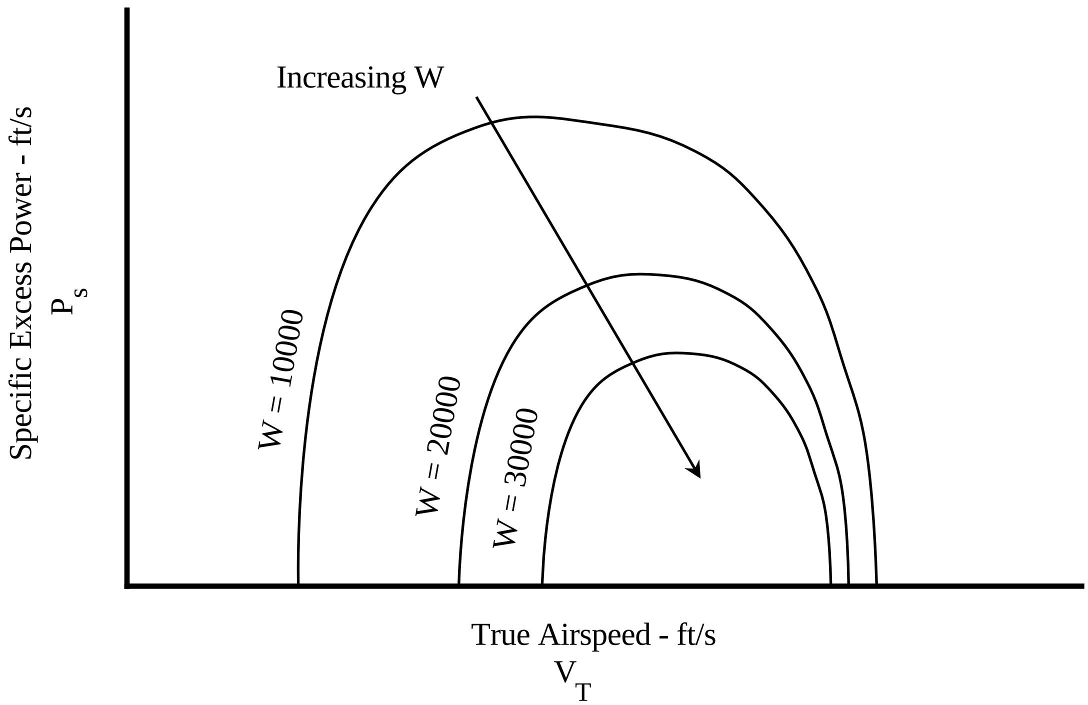

Figure 5.8

EFFECT OF INCREASED GROSS WEIGHT ON SPECIFIC EXCESS POWER

For example, compare the P_s curves for an airplane at 2 g with one at twice the standard weight shown in figure 5.9. At the maximum and minimum level flight speeds where $P_s = 0$, the additional induced drag is the same in both cases. The balance of thrust and drag is the same, resulting in identical minimum and maximum level flight speeds. At intermediate speeds where $P_s > 0$, the value of P_s for the high gross weight case is half that of the aircraft at 2 g even though the actual excess power may be the same (P_s is specific to the higher weight).

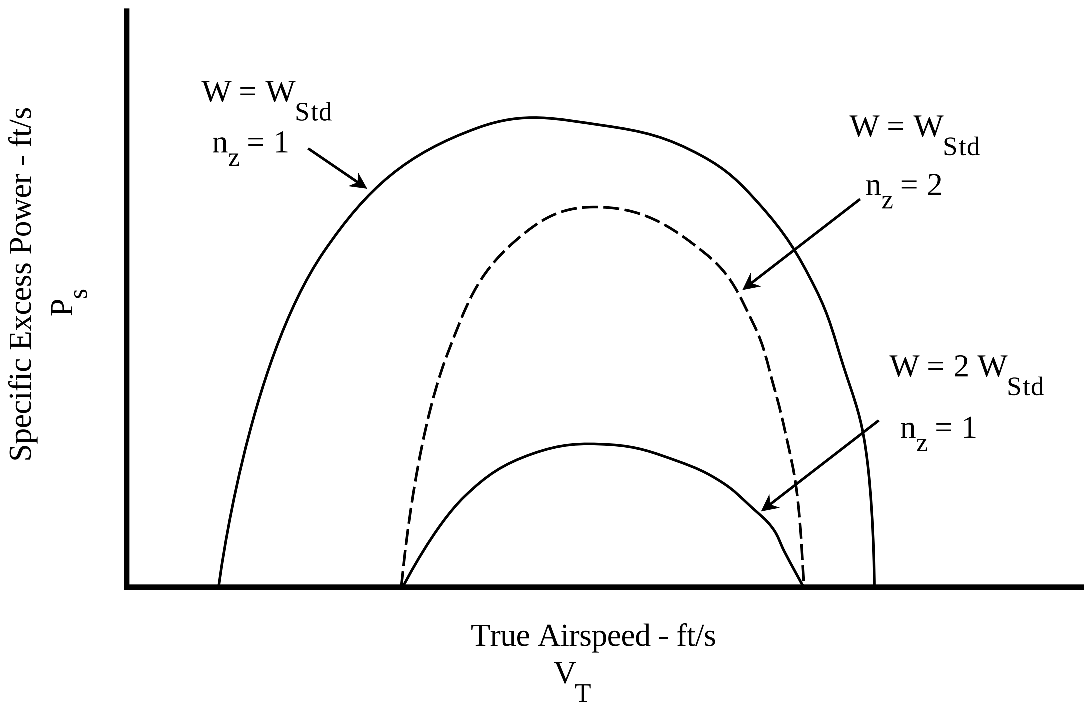

Figure 5.9
COMPARING EFFECT OF INCREASED GROSS WEIGHT WITH INCREASED NORMAL ACCELERATION

5.3.5.3 INCREASED PARASITE DRAG

Increasing the airplane's parasite drag has an effect which increases as airspeed increases. As drag is increased, both P_S and the speed for maximum P_S decrease (Figure 5.10):

$$P_s = \frac{V_T \left(T_{N_x} - D - \Delta D_p \right)}{W} \qquad \text{(Eq 5.22)}$$

Where:

D	Drag	lb
ΔD_p	Change in parasite drag	lb
P_S	Specific excess power	ft/s
T_{N_x}	Net thrust parallel flight path	lb
V_T	True airspeed	ft/s
W	Weight	lb.

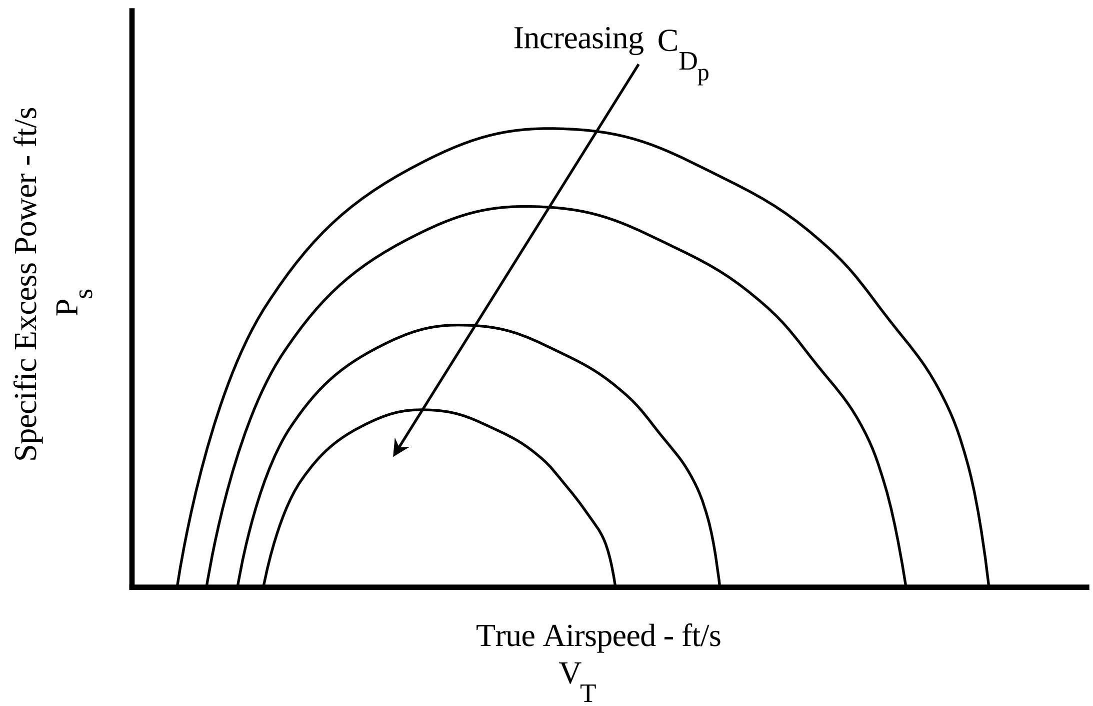

Figure 5.10
EFFECT OF INCREASED DRAG ON SPECIFIC EXCESS POWER

5.3.5.4 INCREASED THRUST

Increasing thrust increases P_S. As thrust is increased, both P_S and the speed for maximum P_S increase (Figure 5.11):

$$P_s = \frac{V_T\,(T_{N_x} + \Delta T_{N_x} - D)}{W} \qquad \text{(Eq 5.23)}$$

Where:

D	Drag	lb
P_S	Specific excess power	ft/s
T_{N_x}	Net thrust parallel flight path	lb
V_T	True airspeed	ft/s
W	Weight	lb.

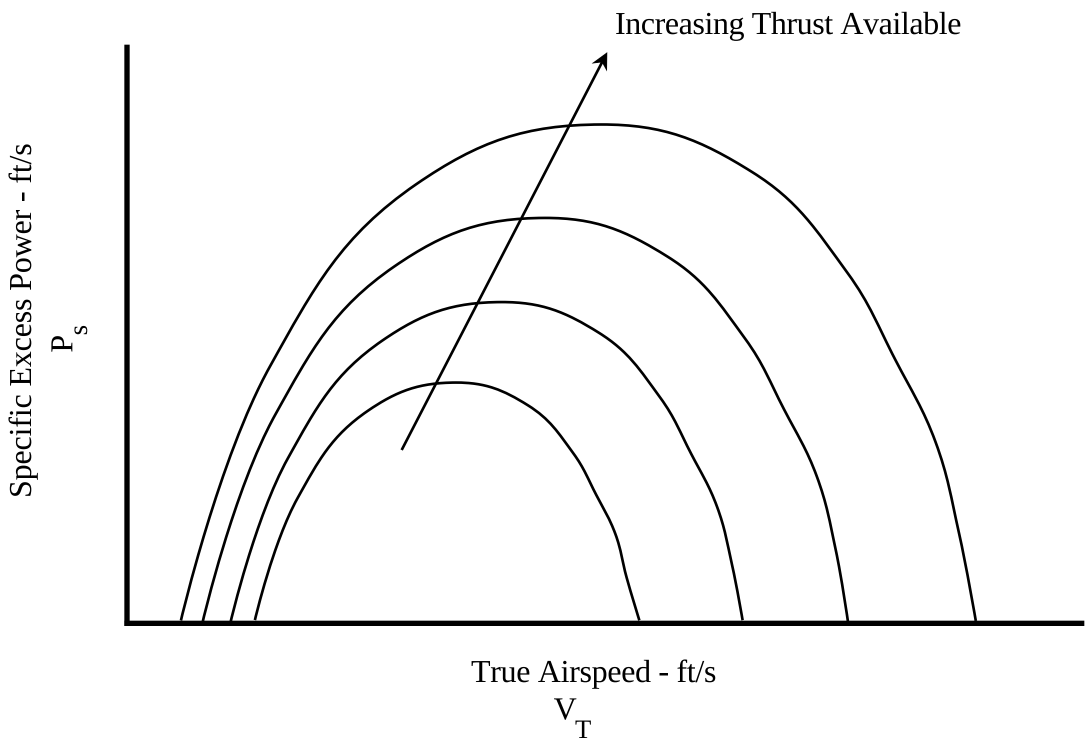

Figure 5.11
EFFECT OF INCREASED THRUST ON SPECIFIC EXCESS POWER

5.3.5.5 INCREASED ALTITUDE

The typical result of an increase in altitude is shown in figure 5.12.

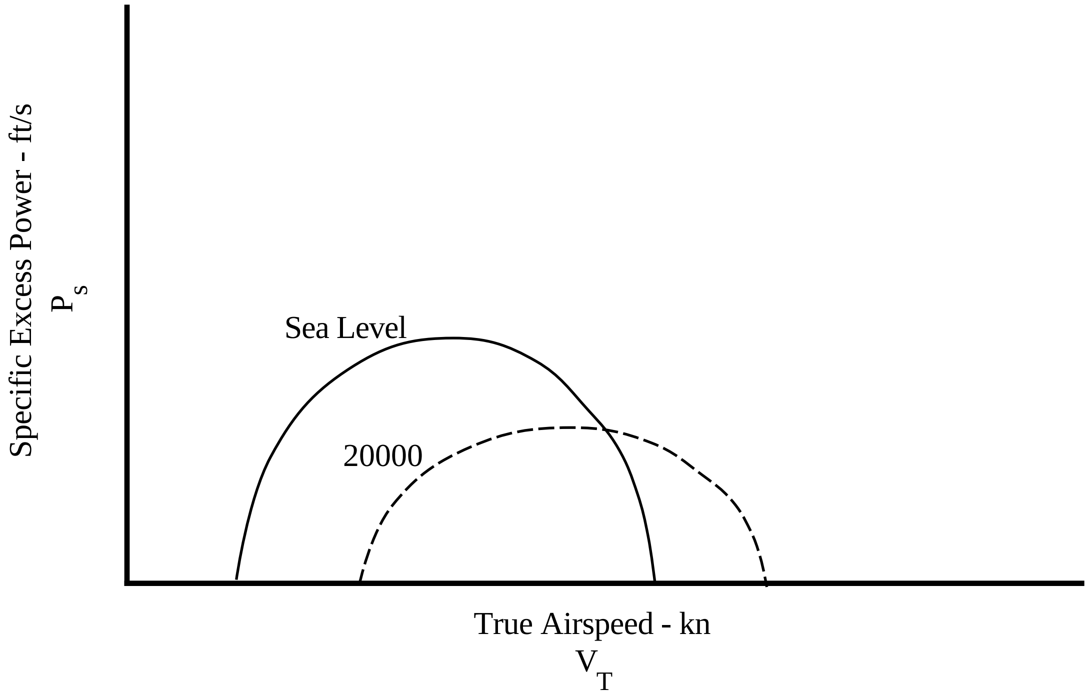

Figure 5.12
EFFECT OF INCREASED ALTITUDE ON SPECIFIC EXCESS POWER

As altitude is increased, both power available and power required are affected. The power required increases with increasing true airspeed. The power available decreases depending on the particular characteristics of the engine:

$$P_s = \frac{\left(P_A - \Delta P_A\right) - \left(P_{req} + \Delta P_{req}\right)}{W} \qquad \text{(Eq 5.24)}$$

Where:

P_A	Power available	ft-lb/s
P_{req}	Power required	ft-lb/s
P_S	Specific excess power	ft/s
W	Weight	lb.

5.3.5.6 SUBSONIC AIRCRAFT

The specific excess power characteristics for subsonic aircraft designs are generally dominated at high speeds by the transonic drag rise. At low altitudes, however, the high dynamic pressures for high subsonic speeds may impose a structural envelope limit which effectively prevents the airplane from reaching its performance potential. These aircraft have to be throttled back for those conditions to avoid damage to the airframe. In general, the transonic drag rise determines the high speed P_S characteristics as shown in figure 5.13.

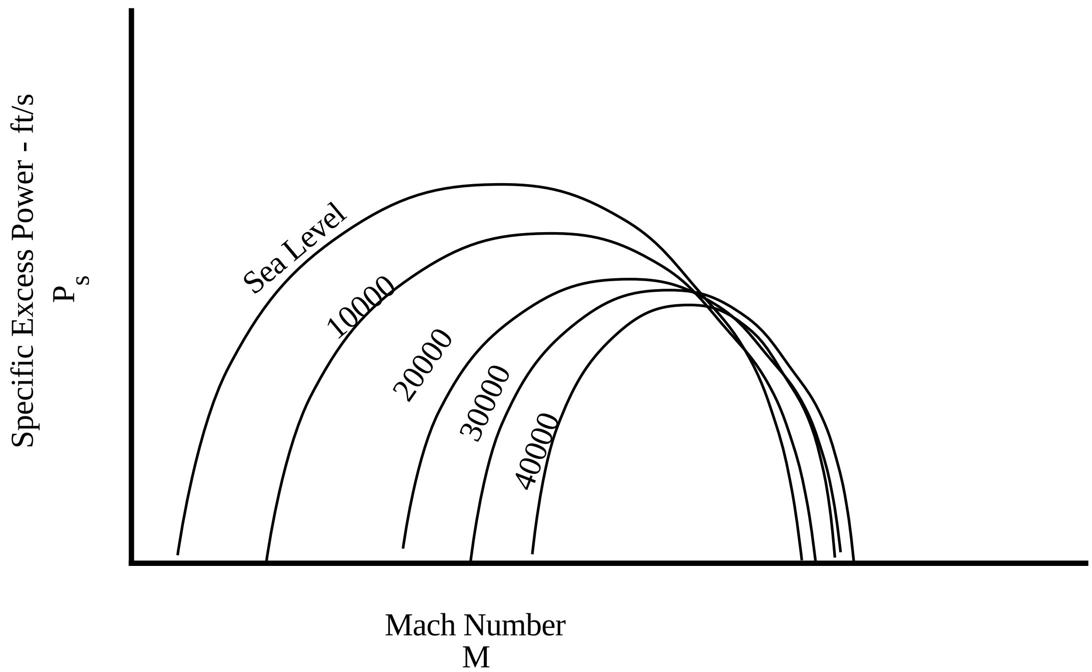

Figure 5.13
TYPICAL SPECIFIC EXCESS POWER FOR SUBSONIC AIRPLANE

5.3.5.7 SUPERSONIC AIRCRAFT

The specific excess power characteristics of a supersonic aircraft takes a form depending on the variation of net thrust and drag with Mach number. There is typically a reduction in P_S in the transonic region resulting from the compressibility drag rise. When P_S is substantially reduced in the transonic region, the level acceleration is slowed very noticeably. The aircraft may require afterburner to accelerate through the transonic drag rise but are then capable of sustaining supersonic flight in military power. The P_S plot for a typical supersonic aircraft is shown in figure 5.14.

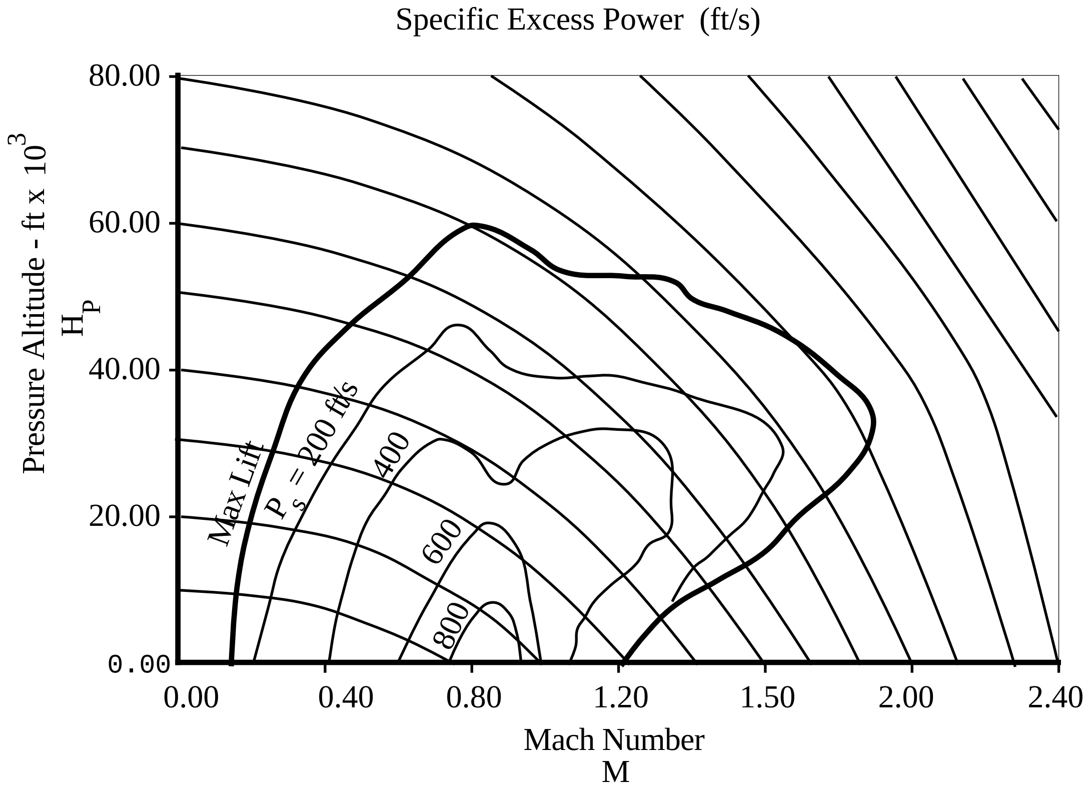

Figure 5.14

TYPICAL SPECIFIC EXCESS POWER SUPERSONIC AIRPLANE

5.4 TEST METHODS AND TECHNIQUES

The principal test method for obtaining P_S data is the level acceleration. The sawtooth climb method is used for cases when P_S is low or the aircraft is limited by gear or flap extension speed in the takeoff or landing configuration. Other methods include the use of extremely sensitive inertial platforms for dynamic test techniques.

Since the P_S analysis distinguishes between increased energy from a climb and increased energy from acceleration, any measurement errors in height (tapeline altitude) degrades the accuracy of the final results. Reliance on pitot static instrumentation, even when specially calibrated, does not produce results as good as can be obtained with more sophisticated, absolute space positioning equipment such as high resolution radar or laser tracking devices.

5.4.1 LEVEL ACCELERATION

The specific excess power characteristics for the entire flight envelope are determined from a series of level accelerations performed at different altitudes, usually separated by about 5,000 ft.

The acceleration run should be started at as low a speed as practicable with the engine(s) stabilized at the desired power setting (usually MIL or MAX). This requirement presents difficulties in flight technique when P_S is high. A commonly used method is to stabilize the aircraft in a climb at the desired speed sufficiently below the target altitude long enough to allow the engine(s) to reach normal operating temperatures. Speedbrakes, and sometimes flaps, are used to increase drag to reduce the rate of climb. As the target altitude is approached, speedbrakes and flaps are retracted and the airplane is pushed over into level flight. The first few seconds of data are discarded usually, but this technique enables a clean start with the least data loss.

If P_S is negative at the minimum airspeed, the start must be above the target altitude in a descending acceleration. The objective is to level at the target altitude at a speed where P_S is positive and the acceleration run can proceed normally.

During the acceleration run, maintain the target altitude as smoothly as possible. Altitude variations are taken into account in the data reduction process and only affect the results when they are large enough to produce measurable changes in engine performance. Changes in induced drag caused by variations in normal acceleration cannot be accounted for, or corrected, and generate significant errors. The altitude may be allowed to vary as much as ± 1000 ft around the target altitude without serious penalties in the accuracy of P_S data but the normal acceleration must be held within ± 0.1 g. Using normal piloting techniques, considerably tighter altitude tolerances are easily achievable without exceeding the g limits. The altitude tolerance is typically ± 300 ft. The normal acceleration tolerance of ± 0.1 g allows the pilot to make shallow turns for navigational purposes during the acceleration run. The g will remain within tolerance if the bank angle does not exceed 10˚, but the turn should be limited to no more than a 30˚ heading change to minimize the build up of errors.

Smoothness during the acceleration is helped by anticipation and attitude flying. If the mechanical characteristics of the longitudinal control system make small precise inputs around trim difficult, the airplane may be flown off trim so force reversals are not encountered during the acceleration. In some aircraft, trimming during the run is inadvisable because the smallest possible trim input can cause an unacceptably large variation in normal acceleration. Others may have trim system characteristics which permit their use. The test aircraft determines the appropriate technique.

Near the maximum level flight airspeed, the P_s approaches zero. The acceleration run is usually terminated when the acceleration drops below a given threshold, usually 2 kn/min. The P_s data are anchored by determining the $P_s = 0$ airspeed, using the front side technique presented in Chapter 4. When the acceleration drops below 2 kn/min, smoothly push over to gain 5 to 10 kn, then smoothly level off. Hold the resulting lower altitude until the airspeed decreases and stabilizes (less than 2 kn/min change) at the maximum level flight airspeed.

5.4.1.1 DATA REQUIRED

The following data are required at intervals throughout the acceleration run:

Time, H_{P_o}, V_o, $\dot{W}_f$, OAT, W_f.

The desired frequency of data recording depends on the acceleration rate. When the acceleration is low, acceptable results can be achieved using manual recording techniques and taking data every few seconds. As the acceleration increases, hand-held data-taking becomes more difficult. For anything more than moderate acceleration rates some form of automatic data recording is essential.

5.4.1.2 TEST CRITERIA

1. Coordinated, level flight during the acceleration run.
2. Engine(s) stabilized at normal operating temperatures.
3. Altimeter set to 29.92.

5.4.1.3 DATA REQUIREMENTS

1. H_{P_o} ± 300 ft.
2. Normal acceleration ± 0.1 g.
3. Bank angle ≤ 10˚.
4. Heading change ≤ 30˚.

5.4.1.4 SAFETY CONSIDERATIONS

There are no unique hazards or safety precautions associated with level acceleration runs. However, take care to observe airspeed limitations and retract flaps or speedbrakes if used to help control the entry to the run.

5.4.2 SAWTOOTH CLIMBS

Sawtooth climbs provide a useful alternative method of obtaining P_S data, especially when P_S is low or there are airspeed limits which must be observed, as in the takeoff, landing, wave-off or single engine configurations. The technique consists of making a series of short climbs (or descents, if P_S is negative at the test conditions) at constant V_o covering the desired range of airspeeds. The altitude band for the climbs is usually the lesser of 1000 ft either side of the target altitude or the height change corresponding to two minutes of climb (or descent).

The same altitude band should be used for each climb, until P_S becomes so low that the climbs are stopped after two minutes, in which case the starting and ending altitudes are noted. The target altitude must be contained within the climb band, preferably close to the middle. As P_S decreases, and time rather than altitude change becomes the test criterion, the climb band shrinks symmetrically about the target altitude.

As with the acceleration runs, sufficient altitude should be allowed for the engine(s) to reach normal operating temperatures and the airplane to be completely stabilized at the desired airspeed before entering the data band. Smoothness is just as important as in the acceleration runs and for the same reasons. The tolerance on airspeed is ± 1 kn, but this must not be achieved at the expense of smoothness. If a small airspeed error is made while establishing the climb, maintaining the incorrect speed as accurately as possible is preferred rather than trying to correct it and risk aborting the entire run. The speed should, of course, be noted. Sawtooth climb test techniques and data reduction are discussed further in Chapter 7.

5.4.3 DYNAMIC TEST METHODS

The modern techniques of performance testing use dynamic test methods. The crucial requirements for dynamic test methods are:

1. Accurate measure of installed thrust in flight.
2. Accelerometers of sufficient sensitivity and precision to enable highly accurate determination of rates and accelerations in all three axes.

The desired objective of dynamic performance testing is to generate accurate C_L/C_D plots similar to the one shown in figure 5.15.

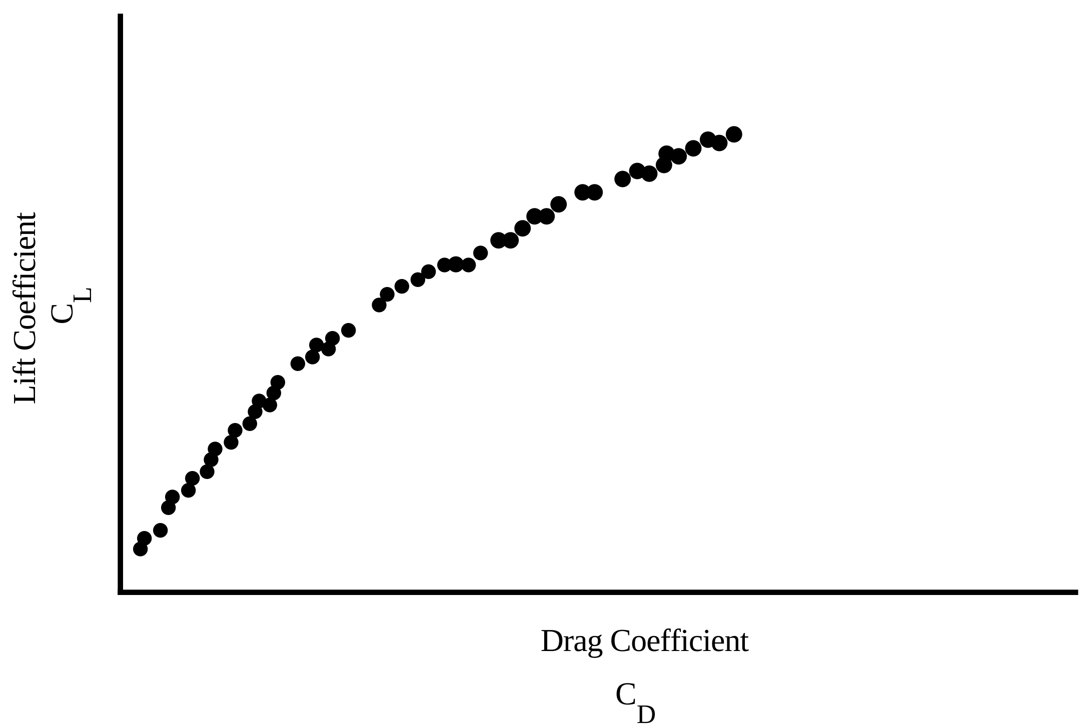

Figure 5.15
LIFT VERSUS DRAG COEFFICIENT DERIVED FROM DYNAMIC PERFORMANCE TESTS

Once these plots have been produced, cruise, turn, and acceleration performance can be modelled using the same validated thrust model used to generate the C_L/C_D plots.

The fundamental theory underlying the generation of these plots is to derive expressions for C_L and C_D in terms of known or measurable quantities (including thrust, weight, x, and z accelerations).

The Pressure Area method, or the Mass Flow method, enable inflight thrust measurements to be performed to accuracies of 3-5% as was demonstrated during the X-29 program, in which eight different telemetered pressure measurements allowed continuous, real-time determination of thrust.

There are three methods of measuring the x and z accelerations: CG (or body axis) accelerometers, flight path accelerometers (FPA) and inertial navigation systems (INS). CG accelerometers are strapped to the airframe and sense accelerations along the three orthogonal body axes. The FPA mounts on a gimballed platform at the end of a nose boom similar to a swivelling pitot head. Accelerations are measured relative to the flight path. Finally, INS may be used to record the accelerations. In this case the INS measurements are taken in the inertial reference frame.

In general, any of these methods generate the values of x and z accelerations required to calculate the values of C_L and C_D. However, various transformations and corrections must be performed depending upon the accelerometer configuration used. The test techniques used in dynamic performance testing include non-steady profiles such as the push-over, pull-up (POPU) (Figure 5.16), the wind-up-turn (WUT), or the split-S (SS).

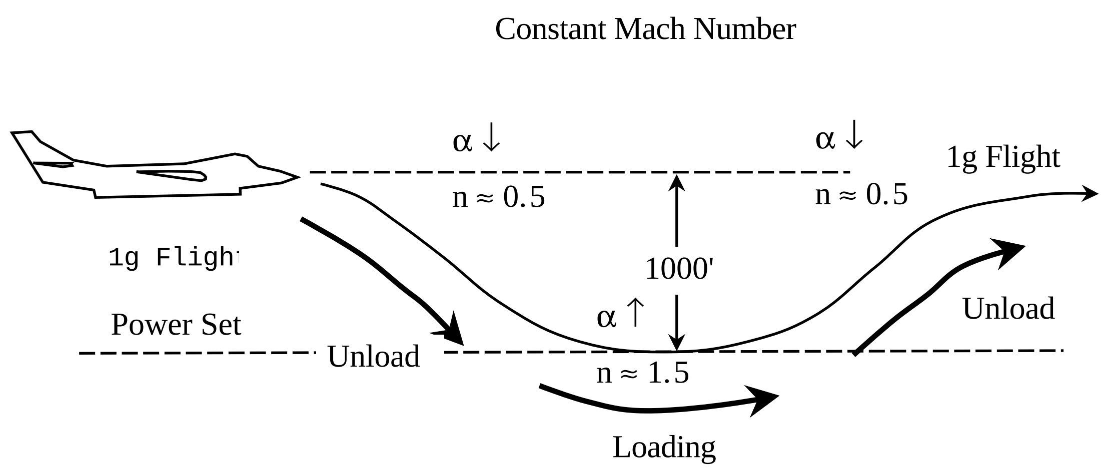

Figure 5.16
PUSH-OVER PULL-UP MANEUVER

The aircraft is flown through a sweep of angle of attack (α), and hence pitch rate, at constant Mach number. Because of the dynamics of the maneuver, some corrections are made. Examples of the type of corrections applied to this data include:

1. Pitch rate corrections to α. Because the aircraft is pitching, a FPA registers an error in the value of α (increase for nose-down pitch rates, decrease for nose-up pitch rates).
2. Accelerometer rate corrections. Placement of the accelerometers at the end of a nose boom means they measure not only the accelerations of the aircraft but accelerations due to the rotation of the aircraft about its CG, and accelerations due to angular accelerations of the aircraft.
3. Local flow corrections. Errors result from the immersion of an FPA in an upwash field ahead of the airplane.
4. Boom bending. An FPA mounted at the end of a boom is subjected to errors caused by bending of the boom under load.
5. Transformation of inertial velocities into accelerations relative to the wind, or stability axes.
6. Transformation of accelerations sensed by CG accelerometers from body axes to stability axes.

The significance of dynamic performance testing methods is the capability to acquire large quantities of data quickly from a single maneuver. A relatively small number of POPUs, WUTs or SSs may be flown instead of a large series of level accelerations, stabilized cruise points, and steady sustained turn performance points. In practice, a number of conventional tests are required to validate the performance model established by the results of the dynamic tests. However, this number is small and decreases as confidence in the technique is gained.

5.5 DATA REDUCTION

5.5.1 LEVEL ACCELERATION

The following equations are used to reduce level acceleration data.

$$q_c = P_{ssl} \left\{ \left[1 + 0.2 \left(\frac{V_c}{a_{ssl}} \right)^2 \right]^{3.5} - 1 \right\} \tag{Eq 5.25}$$

$$P_a = P_{ssl} \left(1 - 6.8755856 \times 10^{-6} H_{P_c} \right)^{5.255863} \tag{Eq 5.26}$$

$$M = \sqrt{ \frac{2}{\gamma - 1} \left[\left(\frac{q_c}{P_a} + 1 \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] } \tag{Eq 5.27}$$

$$T_a = \frac{OAT + 273.15}{1 + \frac{\gamma - 1}{2} K_T M^2} \tag{Eq 5.28}$$

$$h = H_{P_c} \frac{T_{a_{Test}}}{T_{a_{Std}}} \tag{Eq 5.29}$$

$$V_T = M \sqrt{\gamma g_c R T_a} \tag{Eq 5.30}$$

$$E_h = h + \frac{V_T^2}{2\,g} \qquad \text{(Eq 5.7)}$$

Where:

a_{ssl}	Standard sea level speed of sound	661.483 kn
E_h	Energy height	ft
g	Gravitational acceleration	ft/s^2
γ	Ratio of specific heats	
g_c	Conversion constant	32.17 $lb_m/slug$
h	Tapeline altitude	ft
H_{P_c}	Calibrated pressure altitude	ft
K_T	Temperature recovery factor	
M	Mach number	
OAT	Outside air temperature	˚C
P_a	Ambient pressure	psf
P_{ssl}	Standard sea level pressure	2116.217 psf
q_c	Impact pressure	psf
R	Engineering gas constant for air	96.93ft-lb_f/lb_m-˚K
T_a	Ambient temperature	˚K
$T_{a_{Std}}$	Standard ambient temperature	˚K
$T_{a_{Test}}$	Test ambient temperature	˚K
V_c	Calibrated airspeed	kn
V_T	True airspeed	ft/s.

Correct observed altitude and airspeed data to calibrated altitude and airspeed. Using calibrated altitude, airspeed, and OAT compute E_h as follows:

Step	Parameter	Notation	Formula	Units	Remarks
1	Impact pressure	q_c	Eq 5.25	psf	
2	Ambient pressure	P_a	Eq 5.26	psf	
3	Mach number	M	Eq 5.27		
4	Ambient temperature	T_a	Eq 5.28	˚K	Or from reference source
5	Tapeline height	h	Eq 5.29	ft	
6	True airspeed	V_T	Eq 5.30	ft/s	
7	Energy height	E_h	Eq 5.7	ft	

Plot E_h as a function of elapsed time as shown in figure 5.17.

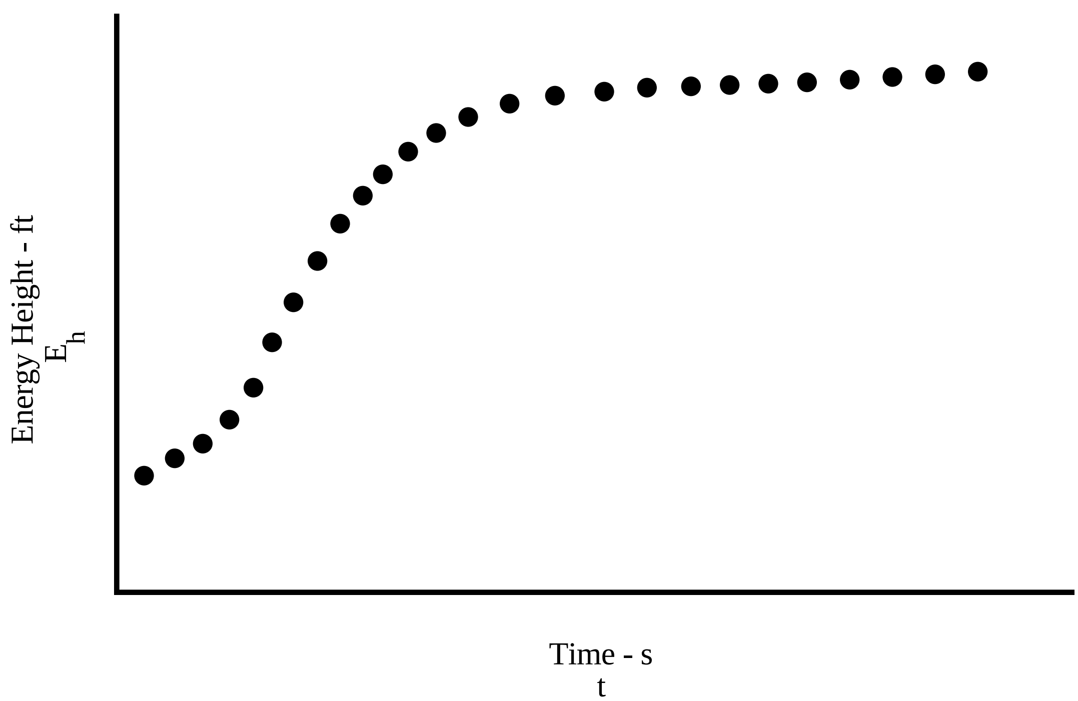

Figure 5.17
ENERGY HEIGHT VERSUS ELAPSED TIME

Fair a curve through the data points of figure 5.17 and find its derivative ($P_s = dE_h/dt$) at a sufficient number of points. Plot P_s against Mach number or true airspeed as in figure 5.18.

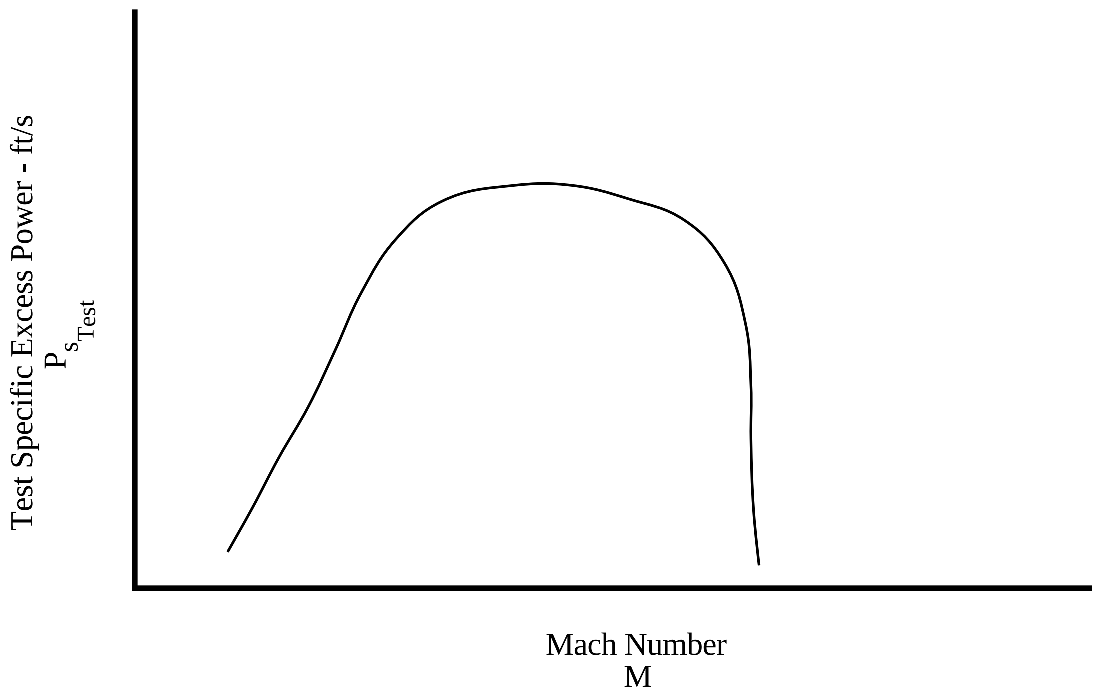

Figure 5.18
TEST SPECIFIC EXCESS POWER VERSUS MACH NUMBER

5.5.2 CORRECTING FOR NON-STANDARD CONDITIONS

P_S values obtained from the level acceleration method reflect the test day conditions and must be generalized to standard weight and standard atmospheric conditions. The following equations are used to correct for:

1. W for non-standard weight.
2. V_T for non-standard temperature.
3. T for temperature effect on thrust.
4. D for induced drag change resulting from the weight correction.

The weight ratio is calculated from the aircraft fuel state and fuel flow data ($\dot{W}_f$):

$$\frac{W_{Test}}{W_{Std}} \qquad \text{(Eq 5.31)}$$

The velocity ratio is determined from:

$$\frac{V_{T_{Std}}}{V_{T_{Test}}} = \frac{M_{Std}\sqrt{\theta_{Std}}}{M_{Test}\sqrt{\theta_{Test}}} \tag{Eq 5.32}$$

For a constant Mach number correction, $M_{Std} = M_{Test}$ so:

$$\frac{V_{T_{Std}}}{V_{T_{Test}}} = \sqrt{\frac{T_{a_{Std}}}{T_{a_{Test}}}} \tag{Eq 5.33}$$

The change in thrust with temperature at constant altitude and constant Mach number is computed from the engine thrust model:

$$\Delta T = f\left(T_a\right) \tag{Eq 5.34}$$

The change in induced drag with gross weight is computed from the aircraft drag model (drag polar). For constant altitude and constant Mach number, parasite drag is constant and for a parabolic drag polar:

$$\Delta D = D_{Std} - D_{Test} = \frac{2\left(W_{Std}^2 - W_{Test}^2\right)}{\pi\, e\, AR\, S\, \gamma\, P_a\, M^2} \tag{Eq 5.35}$$

Eq 5.36 is used to correct $P_{s_{Test}}$ to $P_{s_{Std}}$.

$$P_{s_{Std}} = P_{s_{Test}} \frac{W_{Test}}{W_{Std}} \sqrt{\frac{T_{a_{Std}}}{T_{a_{Test}}}} + \frac{V_{T_{Std}}}{W_{Std}} \left(\Delta T_{N_x} - \Delta D\right) \tag{Eq 5.36}$$

Where:

AR	Aspect ratio	
D_{Std}	Standard drag	lb
D_{Test}	Test drag	lb
e	Oswald's efficiency factor	
M	Mach number	
M_{Std}	Standard Mach number	
M_{Test}	Test Mach number	
π	Constant	
P_a	Ambient pressure	psf
P_s	Specific excess power	ft/s
$P_{s_{Std}}$	Standard specific excess power	ft/s
$P_{s_{Test}}$	Test specific excess power	ft/s
θ_{Std}	Standard temperature ratio	
θ_{Test}	Test temperature ratio	
S	Wing area	ft^2
T_a	Ambient temperature	˚K
$T_{a_{Std}}$	Standard ambient temperature	˚K
$T_{a_{Test}}$	Test ambient temperature	˚K
T_{N_x}	Net thrust parallel flight path	lb
T_{Std}	Standard thrust	lb
T_{Test}	Test thrust	lb
$V_{T_{Std}}$	Standard true airspeed	ft/s
$V_{T_{Test}}$	Test true airspeed	ft/s
W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb.

5.5.3 COMPUTER DATA REDUCTION

Various computer programs are in existence to assist in reduction of performance data. This section contains a brief summary of the assumptions and logic which might be used. The treatment is purposefully generic as programs change over time or new ones are acquired or developed. Detailed instructions for the particular computer or program are assumed to be available.

The purpose of the energy analysis data reduction program is to calculate standard day specific excess power for any maneuver performed at constant power setting (idle or military). For level accelerations, the program plots $P_{s_{Std}}$ versus Mach number for any n_z, and calculates the maximum sustained n_z available. Referred fuel flow available versus OAT is plotted. For climbs and descents, the program calculates fuel, time, and no-wind distance. This section deals with level acceleration runs.

Basic data such as aircraft type, standard gross weight, etc., is entered. For each data point the following information is input data.

1. Time (s).
2. Indicated airspeed (kn).
3. Indicated pressure altitude (ft) (29.92).
4. OAT (˚C) or ambient Temperature (˚K).
5. Fuel flow (lb/h).

The program calculates referred parameters for each data point, and plots energy height versus time as in figure 5.17.

The program calculates P_s by taking the derivative of energy height with respect to time. Therefore, a curve is fitted in some manner to the E_h versus time plot. Since P_s is calculated from the slope of this curve, any slight bends in the curve are magnified when the derivative (slope) is calculated. Care must be taken to fit a smooth, accurate curve through the data.

Following completion of the curve fitting, the program computes and plots standard day P_s versus Mach number as in figure 5.18.

The program calculates and plots fuel, time, and distance for the maneuver. Fuel flow is plotted first as referred fuel flow available versus OAT as in figure 5.19.

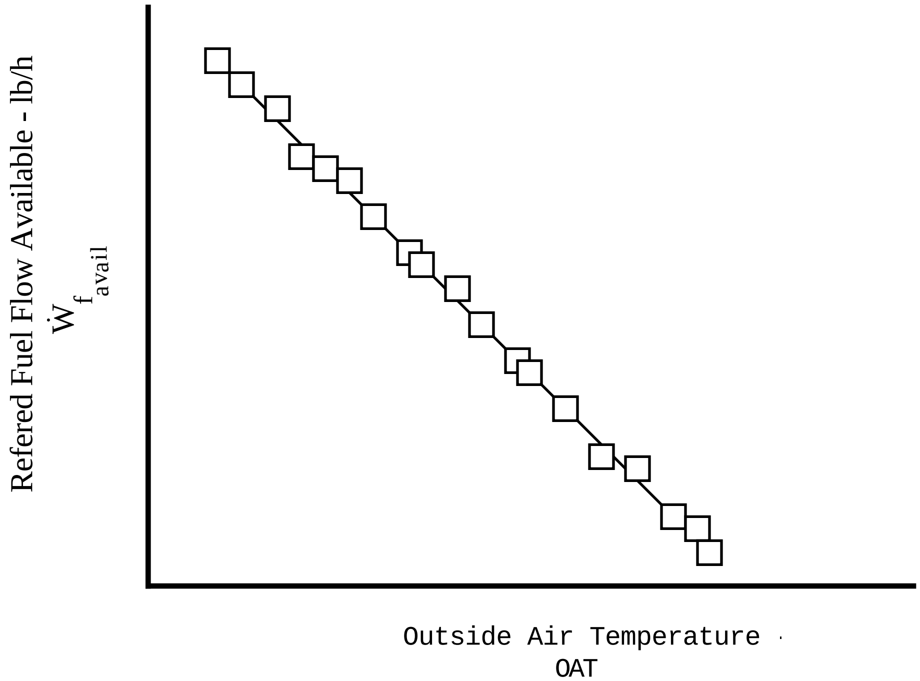

Figure 5.19
REFERRED FUEL FLOW VERSUS OAT

Since fuel flow is referred to total conditions, the curve may be used in combination with range and endurance data to calculate standard day V_H using the method described in Chapter 4.

The program next plots the ratio of $P_{s_{Std}}$ to fuel flow versus Mach number as in figure 5.20.

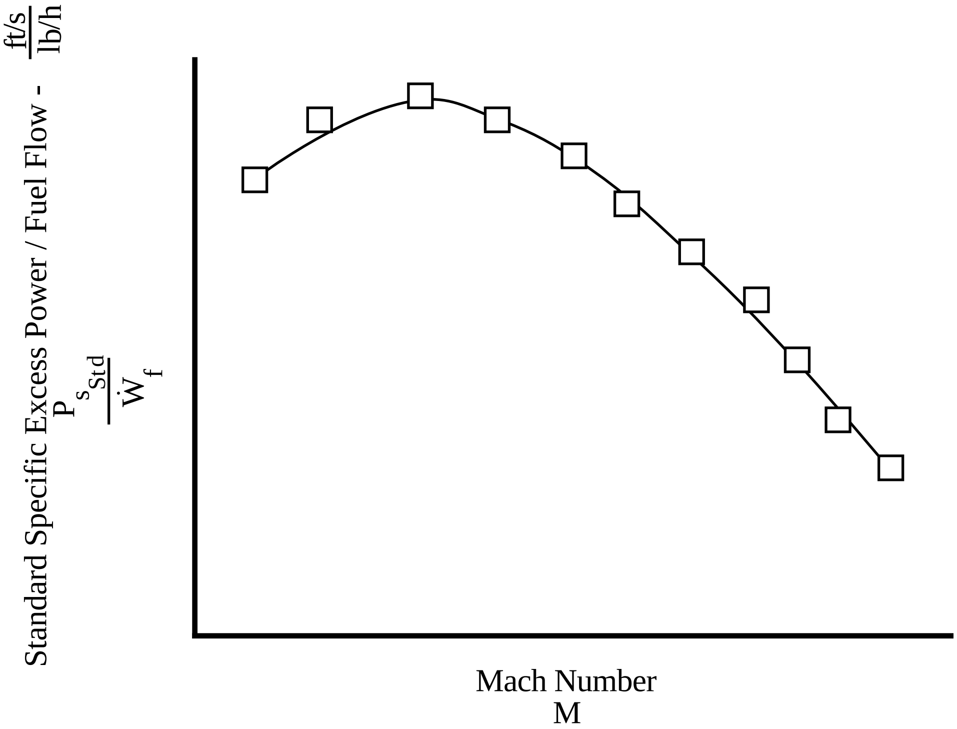

Figure 5.20
STANDARD SPECIFIC EXCESS POWER/FUEL FLOW RATIO VERSUS MACH NUMBER

A family of these plots from several altitudes may be cross-plotted on energy paper and used to determine climb schedules as discussed in Chapter 7.

For turn performance, the program plots $P_{s_{Std}}$ versus Mach number for any n_z, and predicts maximum sustained n_z. Excess power data can be related to turn performance as discussed in Chapter 6.

5.5.3.1 EQUATIONS USED BY THE COMPUTER ROUTINE

Position error:

$$V_c = V_i + \Delta V_{pos} \tag{Eq 5.37}$$

$$H_{P_c} = H_{P_i} + \Delta H_{pos} \tag{Eq 5.38}$$

Mach number:

$$M = f\left(V_c , H_{P_c}\right) \tag{Eq 5.39}$$

Weight:

$$W_{Test} = \text{Initial W} - \int \dot{W}_f \; dt \tag{Eq 5.40}$$

If ambient temperature (˚K) was entered:

$$°C = °K - 273.15 \tag{Eq 5.41}$$

$$OAT = f\left(T_a , M_T\right) \tag{Eq 5.42}$$

If OAT ˚C was entered:

$$T_a = f\left(OAT, M\right) \tag{Eq 5.43}$$

Test day true airspeed:

$$V_{T_{Test}} = f\left(V_c , H_{P_c} , T_a\right) \tag{Eq 5.44}$$

Standard day true airspeed:

$$V_{T_{Std}} = f\left(V_c, H_{P_c}, T_{Std}\right) \tag{Eq 5.45}$$

First data point as $H_{P_{c\,ref}}$:

$$h = H_{P_{c_{ref}}} + \Delta H_{P_c} \left(\frac{T_a}{T_{Std}}\right) \tag{Eq 5.46}$$

Energy height:

$$E_h = h + \frac{V_{T_{Test}}^2}{2g} \quad \text{(Eq 5.47)}$$

Test day P_s from faired E_h versus time curve:

$$P_{s_{Test}} = \frac{dE_h}{dt} \quad \text{(Eq 5.48)}$$

Test day flight path angle, dh/dt from the curve of h versus time:

$$\gamma_{Test} = \sin^{-1}\left(\frac{dh/dt}{V_{T_{Test}}}\right) \quad \text{(Eq 5.49)}$$

Climb correction factor:

$$CCF = 1 + \left(\frac{V_{T_{Std}}}{g}\frac{dV}{dh}\right) \quad \text{(Eq 5.50)}$$

Standard day P_s:

$$P_{s_{Std}} = P_{s_{Test}}\left(\frac{W_{Test}}{W_{Std}}\right)\left(\frac{V_{T_{Std}}}{V_{T_{Test}}}\right) + \left(\frac{V_{T_{Std}}}{W_{Std}}\right)(\Delta T_{N_x} - \Delta D) \quad \text{(Eq 5.51)}$$

$$\left(\frac{dh}{dt}\right)_{Std} = \frac{P_{s_{Std}}}{CCF} \quad \text{(Eq 5.52)}$$

Standard day flight path angle:

$$\gamma_{Std} = \sin^{-1}\left(\frac{\left(\frac{dh}{dt}\right)_{Std}}{V_{T_{Std}}}\right) \quad \text{(Eq 5.53)}$$

The program repeats the $P_{s_{Std}}$ calculation using the new standard until:

$$\left|\gamma_{Test} - \gamma_{Std}\right| < 0.1 \quad \text{(Eq 5.54)}$$

Where:

CCF	Climb correction factor	
D	Drag	lb
ΔH_{pos}	Altimeter position error	ft
ΔV_{pos}	Airspeed position error	kn
E_h	Energy height	ft
g	Gravitational acceleration	ft/s^2
γ_{Std}	Standard flight path angle	deg
γ_{Test}	Test flight path angle	deg
h	Tapeline altitude	ft
H_{P_c}	Calibrated pressure altitude	ft
$H_{P_{c\ ref}}$	Reference calibrated pressure altitude	ft
H_{P_i}	Indicated pressure altitude	ft
M	Mach number	
OAT	Outside air temperature	˚C
P_s	Specific excess power	ft/s
$P_{s_{Std}}$	Standard specific excess power	ft/s
$P_{s_{Test}}$	Test specific excess power	ft/s
T_a	Ambient temperature	˚K
T_{N_x}	Net thrust parallel flight path	lb
T_{Std}	Standard thrust	lb
T_{Test}	Test thrust	lb
V_c	Calibrated airspeed	kn
V_i	Indicated airspeed	kn
$V_{T_{Std}}$	Standard true airspeed	ft/s
$V_{T_{Test}}$	Test true airspeed	ft/s

W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb
$\dot{W}_f$	Fuel flow	lb/h.

5.6 DATA ANALYSIS

The analysis of P_S data is directed towards two objectives. The first is that of determining the optimum climb schedules for the airplane which is discussed in Chapter 7. The second is the evaluation of the airplane's tactical strengths and weaknesses and comparison of those characteristics with potential threat aircraft for which similar data is known.

5.6.1 TACTICAL ANALYSIS

The development of total energy concepts has enabled great progress to be made in analyzing the tactical capability of aircraft. The analysis is especially powerful when flight tests of potential threat aircraft allow direct comparisons to be made between aircraft. Such analysis has been of tremendous help in deciding the most advantageous tactics to be used against different threat aircraft and has led to the inclusion of P_S plots (E-M plots) in tactical manuals. More recently, total energy analysis has played a major part in the development of current research programs in fighter agility.

5.7 MISSION SUITABILITY

Requirements for climb performance will be specified in the detail specification for the aircraft. The determination of mission suitability will depend largely on whether the aircraft meets these requirements, and on the type of analysis described in the previous section. The precise shape of the aircraft's P_S envelopes probably will not be specified, although the shape may be implicit in a requirement. Certain P_S values may be required over a range of speeds and altitudes. The final evaluation of mission suitability will depend on more specific flight tests such as rate of climb and agility testing.

5.8 SPECIFICATION COMPLIANCE

Specification compliance for P_s characteristics is concerned with meeting the requirements of the detailed specification for the aircraft. Published specifications, such as MIL-1797, have general applicability but only in the context of requiring that the flying qualities should allow the performance potential to be achieved by using normal piloting techniques.

5.9 GLOSSARY

5.9.1 NOTATIONS

AR	Aspect ratio	
a_{ssl}	Standard sea level speed of sound	661.483 kn
CCF	Climb correction factor	
CG	Center of gravity	
D	Drag	lb
ΔD_i	Change in induced drag	lb
ΔD_p	Change in parasite drag	lb
ΔH_{pos}	Altimeter position error	ft
D_{Std}	Standard drag	lb
D_{Test}	Test drag	lb
ΔV_{pos}	Airspeed position error	kn
e	Oswald’s efficiency factor	
E_h	Energy height	ft
FPA	Flight path accelerometer	
F_x	Forces parallel to flight path	lb
g	Gravitational acceleration	ft/s^2
g_c	Conversion constant	32.17 lb_m/slug
h	Tapeline altitude	ft
H_{P_c}	Calibrated pressure altitude	ft
$H_{P_{c\,ref}}$	Reference calibrated pressure altitude	ft
H_{P_i}	Indicated pressure altitude	ft
INS	Inertial navigation system	
KE	Kinetic energy	ft-lb

K_T	Temperature recovery factor	
M	Mach number	
MAX	Maximum power	
MIL	Military power	
M_{Std}	Standard Mach number	
M_{Test}	Test Mach number	
n_z	Normal acceleration	g
OAT	Outside air temperature	˚C
P_A	Power available	ft-lb/s
P_a	Ambient pressure	psf
PE	Potential energy	ft-lb
POPU	Push-over, pull-up	
P_{req}	Power required	ft-lb/s
P_s	Specific excess power	ft/s
P_{ssl}	Standard sea level pressure	2116.217 psf
$P_{s_{Std}}$	Standard specific excess power	ft/s
$P_{s_{Test}}$	Test specific excess power	ft/s
q_c	Impact pressure	psf
R	Engineering gas constant for air	96.93ft-lb_f/lb_m-˚K
S	Wing area	ft^2
SS	Split-S	
T	Temperature	˚C, or ˚K
	Thrust	lb
t	Time	s
T_a	Ambient temperature	˚K
$T_{a_{Std}}$	Standard ambient temperature	˚K
$T_{a_{Test}}$	Test ambient temperature	˚K
TE	Total energy	ft-lb
T_{N_x}	Net thrust parallel flight path	lb
T_{Std}	Standard thrust	lb
T_{Test}	Test thrust	lb
V	Velocity	ft/s
V_c	Calibrated airspeed	kn
V_i	Indicated airspeed	kn
V_o	Observed airspeed	kn

V_T	True airspeed	kn, ft/s
$V_{T_{Std}}$	Standard true airspeed	ft/s
$V_{T_{Test}}$	Test true airspeed	ft/s
W	Weight	lb
W_f	Fuel weight	lb
W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb
WUT	Wind-up-turn	
$\dot{W}_f$	Fuel flow	lb/h

5.9.2 GREEK SYMBOLS

α (alpha)	Angle of attack	deg
α_j	Thrust angle	deg
γ (gamma)	Flight path angle	deg
	Ratio of specific heats	
γ_{Std}	Standard flight path angle	deg
γ_{Test}	Test flight path angle	deg
π (pi)	Constant	
θ_{Std} (theta)	Standard temperature ratio	
θ_{Test}	Test temperature ratio	

5.10 REFERENCES

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3. USAF Test Pilot School, *Performance Phase Textbook Volume I*, USAF-TPS-CUR-86-01, USAF, Edwards AFB, CA, April, 1986.