CHAPTER 7

CLIMB PERFORMANCE

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CLIMB PERFORMANCE

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$\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt}$	(Eq 7.3)	7.2
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$$V_v = V_T \sin\gamma$$ (Eq 7.18) 7.14

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$$\text{Rate of Climb} = P_s \left(\frac{1}{1 + \frac{V}{g}\frac{dV_T}{dh}}\right)$$ (Eq 7.20) 7.20

$$\text{Time to Climb} = \int_{h_1}^{h_2} \frac{\left(1 + \frac{V}{g}\frac{dV_T}{dh}\right)}{P_s} dh$$ (Eq 7.21) 7.20

$$dt = \frac{dh + \frac{V}{g} dV_T}{P_s}$$ (Eq 7.22) 7.21

$$dE_h = dh + \frac{V}{g} dV$$ (Eq 7.23) 7.21

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$$\text{Fuel to Climb} = \int_{W_1}^{W_2} dW = -\int_{E_{h_1}}^{E_{h_2}} \frac{1}{\frac{dE_h}{dW}}\, dE_h$$ (Eq 7.27) 7.25

$$-\frac{1}{\frac{dE_h}{dW}} = -\frac{dW}{dE_h} = -\frac{dW}{dt}\,\frac{dt}{dE_h} = \frac{\dot{W}_f}{P_s}$$ (Eq 7.28) 7.25

$$\text{Fuel to Climb} = \int_{E_{h_1}}^{E_{h_2}} \frac{\dot{W}_f}{P_s}\, dE_h = \int_{E_{h_1}}^{E_{h_2}} \frac{1}{\frac{P_s}{\dot{W}_f}}\, dE_h$$ (Eq 7.29) 7.26

$$E_{h_{Test}} = h_{Test} + \frac{V_{T_{Test}}^2}{2\,g}$$ (Eq 7.30) 7.37

$$\left(\frac{dh}{dt}\right)_{Test} = P_{s_{Test}}\left(\frac{1}{1 + \frac{V_{T_{ref}}}{g}\,\frac{dV_T}{dh}}\right)$$ (Eq 7.31) 7.38

$$P_{s_{Std}} = P_{s_{Test}}\,\frac{W_{Test}}{W_{Std}}\,\frac{V_{T_{Std}}}{V_{T_{Test}}} + \frac{V_{T_{Std}}}{W_{Std}}\,(\Delta T_{N_x} - \Delta D)$$ (Eq 7.32) 7.38

$$\Delta D = D_{Std} - D_{Test} = \frac{2\left(W_{Std}^2 - W_{Test}^2\right)}{\pi\, e\, AR\, S\, \gamma\, P_a\, M^2}$$ (Eq 7.33) 7.39

$L = n_z W$	(Eq 7.34)	7.40
$n_z = \cos\gamma$	(Eq 7.35)	7.40
$\Delta D = \dfrac{2\left(W_{Std}^2 \cos^2\gamma_{Std} - W_{Test}^2 \cos^2\gamma_{Test}\right)}{\pi\, e\, AR\, \rho_{ssl} V_e^2 S}$	(Eq 7.36)	7.40
$\Delta D = \dfrac{2\left(W_{Std}^2 n_{z\,Std}^2 - W_{Test}^2\, n_{z\,Test}^2\right)}{\pi\, e\, AR\, \rho_{ssl} V_e^2 S}$	(Eq 7.37)	7.40
$\left(\dfrac{dh}{dt}\right)_{Std} = P_{s_{Std}}\left(\dfrac{1}{1 + \dfrac{V_T}{g}\dfrac{dV_T}{dh}}\right)$	(Eq 7.38)	7.41
$\gamma_{Std} = \sin^{-1}\left(\dfrac{\frac{dh}{dt}}{V_T}\right)$	(Eq 7.39)	7.41
$L = W\cos\gamma - T_G \sin\alpha_j$	(Eq 7.40)	7.42
$n_z = \dfrac{L}{W}$	(Eq 7.41)	7.42
$n_z = \cos\gamma - \dfrac{T_G}{W}\sin\alpha_j$	(Eq 7.42)	7.42
$\Delta T_N = f\left(\Delta H_P, \Delta T_a\right)$	(Eq 7.43)	7.44
$\text{Distance} = \int_{t_1}^{t_2} V_T \cos\gamma\, dt$	(Eq 7.44)	7.46
$\text{Fuel Used} = \int_{t_1}^{t_2} \dot{W}_f\, dt$	(Eq 7.45)	7.47
$^\circ C = {}^\circ K - 273.15$	(Eq 7.46)	7.52

$OAT = f\left(T_a, M\right)$	(Eq 7.47)	7.52
$T_a = f\left(OAT, M\right)$	(Eq 7.48)	7.52
$V_T = f\left(OAT, M_T\right)$	(Eq 7.49)	7.53
$h = H_{P_{c\,ref}} + \Delta H_{P_c}\left(\frac{T_a}{T_{std}}\right)$	(Eq 7.50)	7.53

CHAPTER 7

CLIMB PERFORMANCE

7.1 INTRODUCTION

Climb performance is evaluated in various ways depending on the aircraft mission. An interceptor launching to take over a particular combat air patrol (CAP) station is primarily interested in climbing to altitude with the minimum expenditure of fuel. If launching on an intercept, the desire is to reach intercept altitude with the best fighting speed or energy in the minimum time. An attack aircraft launching on a strike mission is primarily interested in climbing on a schedule of maximum range per pound of fuel used. Different missions may require optimization of other factors during the climb. Primary emphasis in this chapter is on energy analysis since measuring climb performance in jet aircraft and determining climb performance for various climb schedules is best done through energy methods. This chapter also discusses the sawtooth climb as an alternate test method of determining specific excess power. However, the sawtooth climb method can be used successfully for lower performance aircraft during climb speed determination and is also suited for single engine climb performance evaluation in the takeoff or wave-off configuration.

7.2 PURPOSE OF TEST

The purpose of this test is to determine the following climb performance characteristics:

1. Conditions for best climb angle.
2. Conditions for best climb rate.
3. Conditions for the shortest time to climb.
4. Conditions for minimum fuel used to climb.
5. Climb schedules for the above conditions.
6. Evaluate the requirements of pertinent military specifications.

7.3 THEORY

7.3.1 SAWTOOTH CLIMBS

The primary method of determining specific excess power (P_s) is the level acceleration. Sawtooth climb is a secondary method of determining P_s. The sawtooth climb is more time consuming than the level acceleration run. There are conditions where the sawtooth climb is more applicable, especially in determining single engine wave-off for multiengine airplanes or climb in the approach configuration for a jet. The sawtooth climb is one method of obtaining the airspeed schedule for maximum rate of climb. The results of tests are derived from a series of short, timed climbs through the same altitude band. The test provides limited information on overall climb performance but does establish the best airspeed at which to climb at a specific altitude.

To express rate of climb potential in terms of P_s, true airspeed (V_T) must be evaluated in the climb. P_s is calculated by measuring energy height as in Eq 7.1 then taking the time derivative as in Eq 7.2.

$$E_h = h + \frac{V_T^2}{2\,g} \qquad \text{(Eq 7.1)}$$

$$P_s = \frac{dE_h}{dt} = \frac{dh}{dt} + \frac{V_T}{g}\frac{dV_T}{dt} \qquad \text{(Eq 7.2)}$$

The only time P_s is equal to rate of climb is when the climb is done at a constant true airspeed, which is rarely the case. If true airspeed is constant, then dV_T/dt in Eq 7.2 is zero. Climbing at constant indicated airspeed is not constant V_T. Climbing at constant Mach number is not constant V_T either, except when climbing in the stratosphere on a standard day.

From the chain rule, dV/dt can be expressed as in Eq 7.3 and substituted into Eq 7.1 to derive Eq 7.4 and Eq 7.5.

$$\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} \qquad \text{(Eq 7.3)}$$

$$P_s = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dh}\frac{dh}{dt} \qquad \text{(Eq 7.4)}$$

$$P_s = \frac{dh}{dt}\left[1 + \frac{V}{g}\frac{dV}{dh}\right] \qquad \text{(Eq 7.5)}$$

Knowing P_S and the climb schedule, the rate of climb potential is:

$$\frac{dh}{dt} = P_s\left[\frac{1}{1 + \frac{V_T}{g}\frac{dV}{dh}}\right] \qquad \text{(Eq 7.6)}$$

Specific excess power is nearly independent of the climb or acceleration path for modest climb angles, but the actual rate of climb is adjusted by the velocity change along the climb path (dV_T/dh). A term called the climb correction factor (CCF) represents the values within the brackets of Eq 7.6 and is useful in evaluating rate of climb. CCF is defined as follows:

$$CCF = \frac{1}{1 + \frac{V_T}{g}\frac{dV}{dh}} \qquad \text{(Eq 7.7)}$$

Where:

CCF	Climb correction factor	
E_h	Energy height	ft
g	Gravitational acceleration	ft/s^2
h	Tapeline altitude	ft
P_s	Specific excess power	ft/s^2
t	Time	s
V	Velocity	ft/s
V_T	True airspeed	ft/s.

There are three cases to consider about the CCF.

1. If the aircraft is accelerating during the climb as in a constant indicated airspeed schedule, CCF < 1 and rate of climb is less than P_S.

2. If the aircraft is climbing at a constant true airspeed, CCF =1 and rate of climb equals P_s.

3. If the aircraft is decelerating while climbing, as in a constant Mach number through decreasing temperature, CCF > 1 and rate of climb is greater than P_s.

For a low speed aircraft the factor does not have much significance. For supersonic aircraft however, the influence of the CCF on rate of climb is significant. Figure 7.1 illustrates CCF as a function of Mach number.

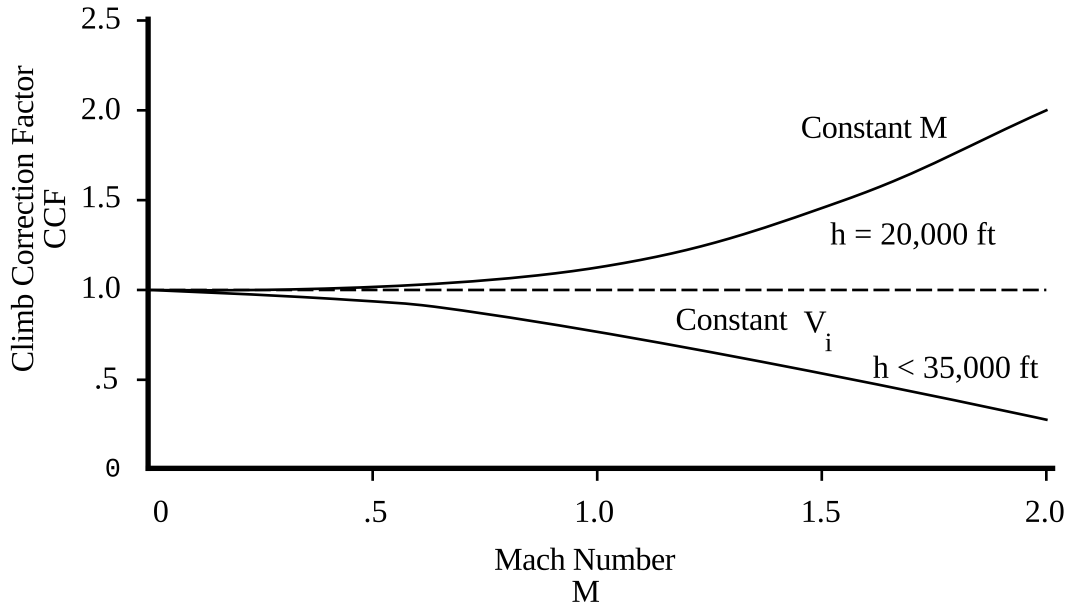

Figure 7.1
CLIMB CORRECTION FACTOR

7.3.2 STEADY STATE APPROACH TO CLIMB PERFORMANCE

The classical approach to aircraft climb performance problems was to use the static or steady state case. One major assumption was the aircraft had no acceleration along the flight path. True airspeed had to be held constant. This approach was derived before the total energy theory was developed, and is inadequate for analyzing climb profiles of supersonic aircraft where both true airspeed and altitude change rapidly. The following paragraphs are intended to present a quick overview of the classical approach.

7.3.2.1 FORCES IN FLIGHT

The forces acting on an aircraft in a climb are presented in figure 7.2 for review and can be resolved perpendicular and parallel to the flight path.

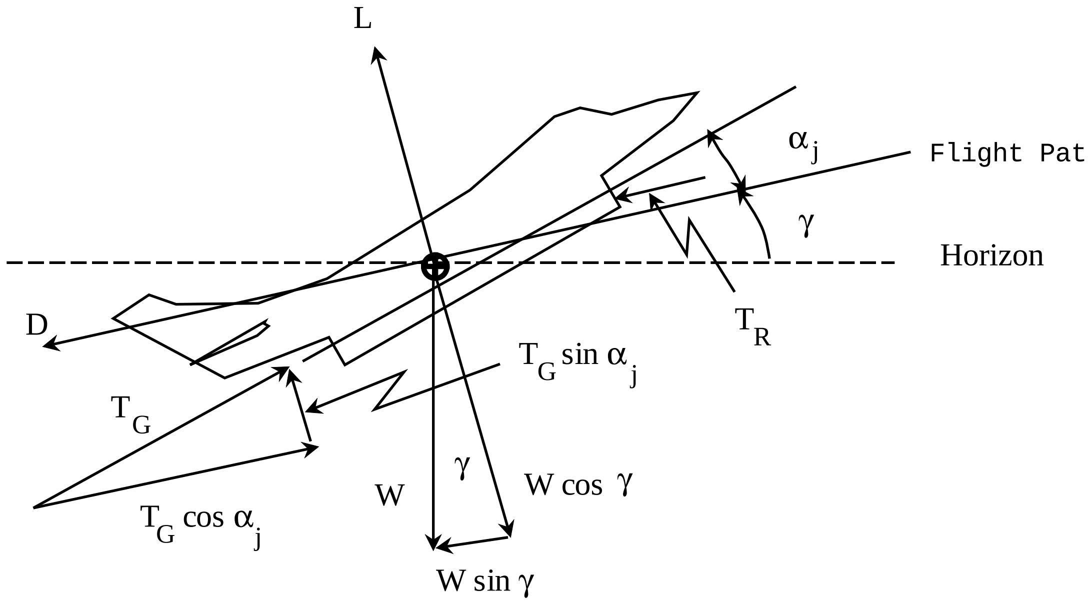

Figure 7.2
FORCES IN CLIMBING FLIGHT

Forces perpendicular to the flight path are:

$$L - W \cos \gamma + T_G \sin \alpha_j = \frac{W}{g} a_z \qquad \text{(Eq 7.8)}$$

Forces parallel to the flight path are:

$$T_G \cos \alpha_j - T_R - D - W \sin \gamma = \frac{W}{g} a_x \qquad \text{(Eq 7.9)}$$

By assuming angle of attack (α) is small, the engines are closely aligned with the fuselage reference line, and the aircraft is in a steady climb where acceleration parallel flight path (a_x) is zero, or at a constant true airspeed, the equations reduce to:

$$L = W \cos \gamma \qquad \text{(Eq 7.10)}$$

$$T_G \cos \alpha_j - T_R - D - W \sin \gamma = \frac{W}{g} \frac{dV_T}{dt} = 0 \qquad \text{(Eq 7.11)}$$

$$T_{N_x} = T_G \cos \alpha_j - T_R \qquad \text{(Eq 7.12)}$$

$$T_{N_x} - D = W \sin \gamma \qquad \text{(Eq 7.13)}$$

With true airspeed held constant, a useful expression for γ can be found:

$$\gamma = \sin^{-1} \left[\frac{T_{N_x} - D}{W} \right] \qquad \text{(Eq 7.14)}$$

The term in brackets is specific excess thrust. By maximizing specific excess thrust, the climb angle is greatest. If both sides of Eq 7.13 are multiplied by V an expression for rate of climb is developed, as Eq 7.15, and is graphically shown in figure 7.3.

$$V \sin \gamma = \frac{\left[T_{N_x} - D \right]}{W} V = \frac{dh}{dt} = V_v \qquad \text{(Eq 7.15)}$$

Where:

α	Angle of attack	deg
α_j	Thrust angle	deg
a_x	Acceleration parallel flight path	ft/s^2
a_z	Acceleration perpendicular to flight path	ft/s^2
D	Drag	lb
dh/dt	Rate of climb	ft/s
γ	Flight path angle	deg
L	Lift	lb
T_G	Gross thrust	lb
T_{N_x}	Net thrust parallel flight path	lb
T_R	Ram drag	lb
V	Velocity	ft/s
V_T	True airspeed	ft/s
V_v	Vertical velocity	ft/s
W	Weight	lb.

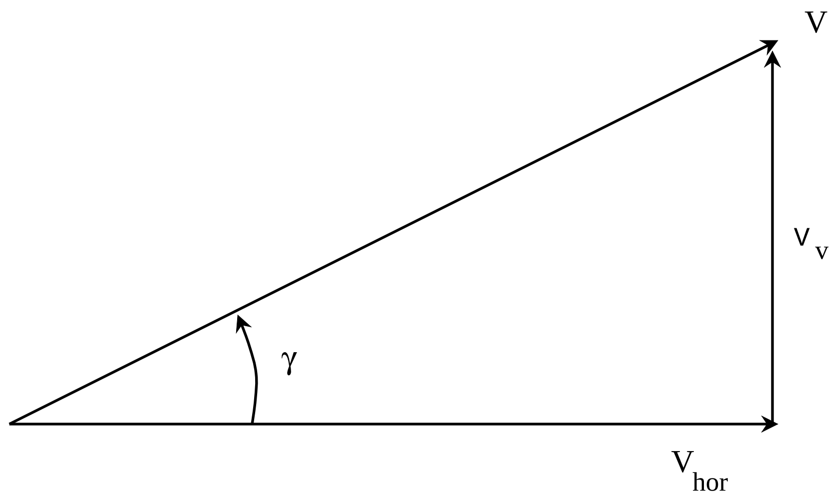

Figure 7.3
CLIMB VECTORS

Eq 7.15 shows if net thrust is greater than drag, dh/dt is positive and a climb results.

7.3.2.2 CLIMB ANGLE

As seen in Eq 7.14, the climb angle, γ, depends on specific excess thrust ($(T_{N_x}$ - D)/W. As the aircraft climbs, the propulsive thrust decreases. Drag remains essentially constant. There is an absolute ceiling where $T_{N_x} = D$ and $\gamma = 0$. Increasing altitude decreases specific excess thrust and the climb angle.

The effect of increasing weight on climb angle can be evaluated from Eq 7.14. Since climb angle is inversely proportional to weight, climb angle decreases as weight increases.

Wind is also a factor. A steady wind has no effect on the climb angle of the aircraft relative to the moving air mass. However, the maximum climb angle must give the most altitude gained for horizontal distance covered. The prime reason for optimizing climb angle might be to gain obstacle clearance during some portion of the flight. Winds do affect this distance and give apparent changes in γ as depicted in figure 7.4.

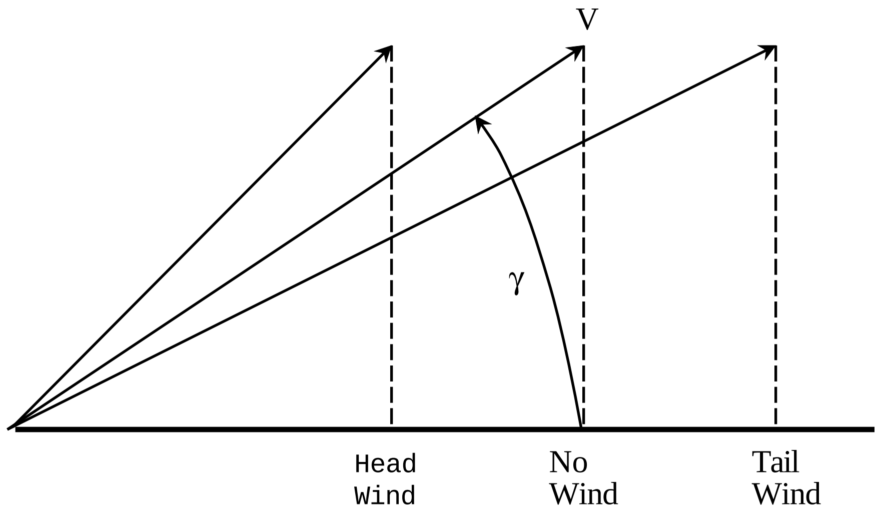

Figure 7.4

WIND EFFECT ON CLIMB ANGLE

Excess thrust, T_{N_x} - D, is a function of airspeed. The aircraft must be flown at the velocity where maximum excess thrust occurs to achieve the maximum climb angle. The net thrust available from a pure turbojet varies little with airspeed at a given altitude. In general, the same can be said for a turbofan. A jet lacking thrust augmentation usually climbs at the velocity for minimum drag or minimum thrust required to achieve the maximum climb angle. This is a classical result and leads to the assumption γ_{max} occurs at $V_{L/D_{max}}$. But, γ_{max} may occur over an airspeed band. In figure 7.5, the maximum excess thrust at military thrust occurs close to $V_{L/D_{max}}$. With thrust augmentation, thrust available varies with airspeed and γ_{max} occurs at other airspeeds.

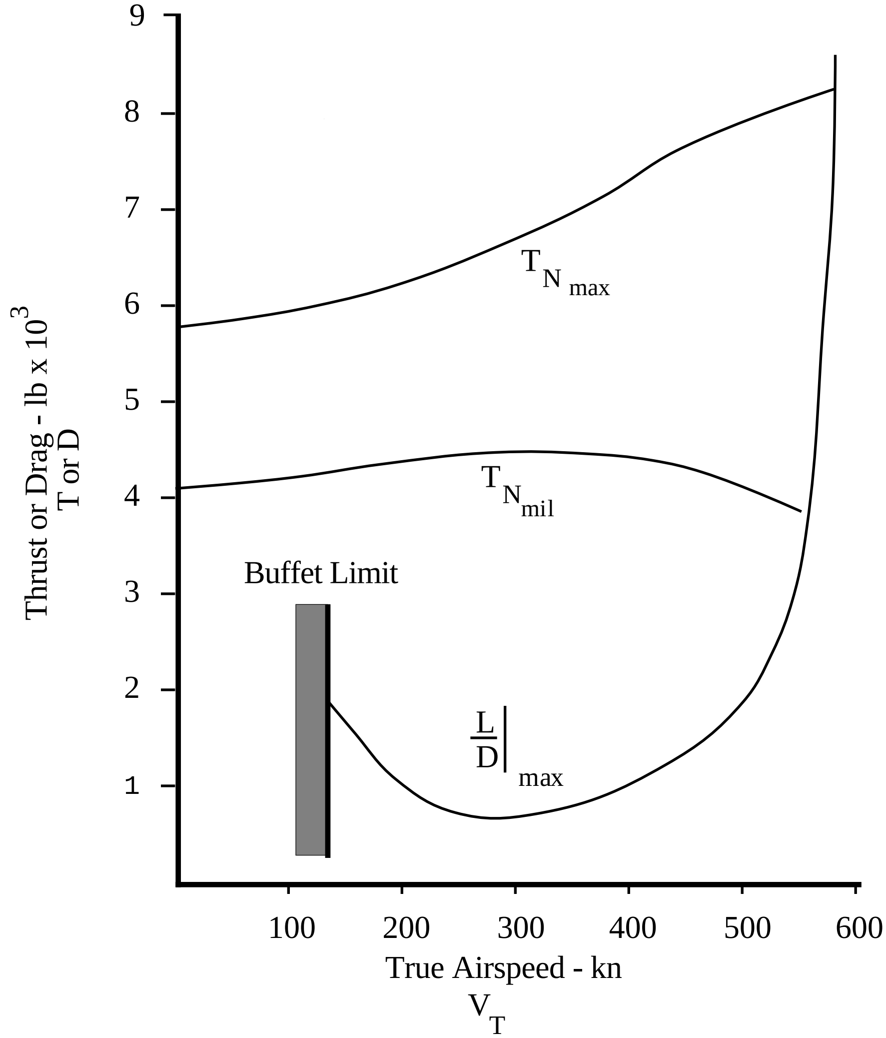

Figure 7.5
THRUST AND DRAG

Specific excess thrust determines climb angle, whether specific excess thrust is measured directly or calculated from independent estimates of thrust, drag, and weight.

7.3.2.3 CLIMB GRADIENT

Climb gradient is the altitude gained for the distance traveled. The gradient can be determined from figure 7.3 by dividing V_v by V_{hor} or by measuring tan γ. The gradient is usually expressed in percent where 100% occurs when $V_v = V_{hor}$, or tan $\gamma = 1$ (45 deg).

7.3.2.4 RATE OF CLIMB

Eq 7.2 shows rate of climb, dh/dt, depends upon specific excess power. This is similar to excess thrust, which was defined as the difference between net thrust available and drag (thrust required) at a specific weight. Excess power is also defined as the difference between the power available to do work in a unit of time and the work done by drag per unit time. Power available can be expressed as net thrust times velocity (T_{N_x} x V) and power lost to drag as drag times velocity (D x V). Then Eq 7.15 can be rewritten as Eq 7.16.

$$ROC = \frac{dh}{dt} = \frac{\left[T_{N_x} - D\right]}{W} V = \frac{T_{N_x} V - D V}{W} = \frac{P_A - P_{req}}{W} \qquad \text{(Eq 7.16)}$$

Where:

D	Drag	lb
dh/dt	Rate of climb	ft/s
T_{N_x}	Net thrust parallel flight path	lb
P_A	Power available	ft-lb/s
P_{req}	Power required	ft-lb/s
ROC	Rate of climb	ft/s
V	Velocity	ft/s, kn
W	Weight	lb.

In specific excess power, dh/dt is rate of climb at constant airspeed. Figure 7.6 depicts a typical curve of power available and required.

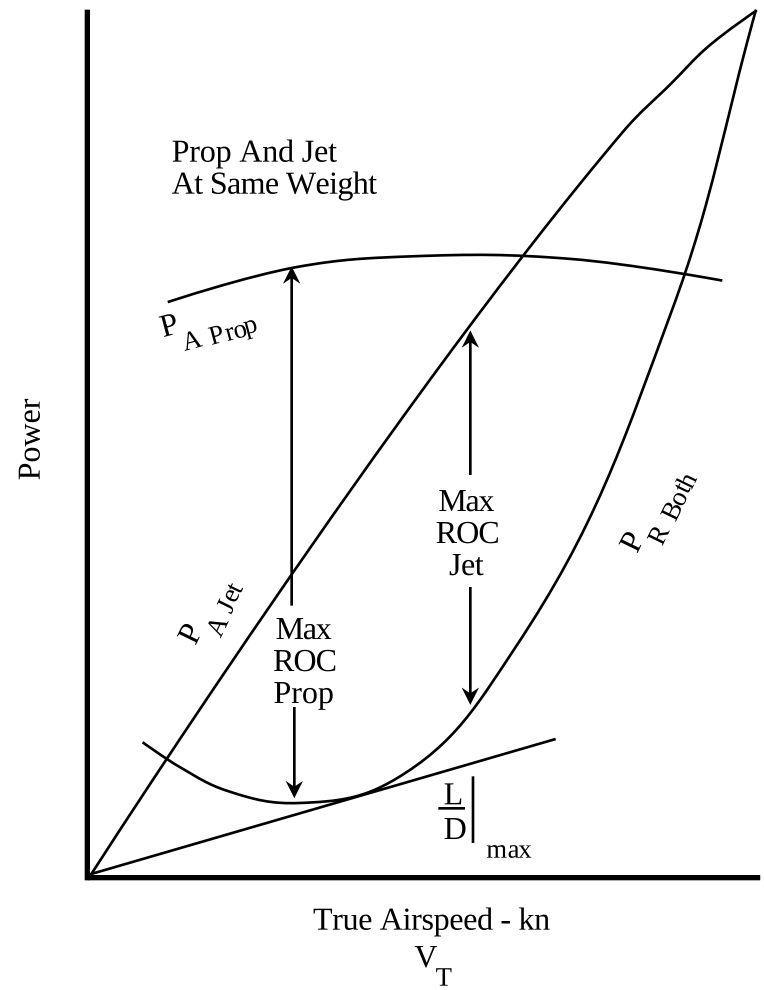

Figure 7.6

POWER AVAILABLE AND REQUIRED

The power required curve is derived by multiplying V by the drag at that speed. The power available curve is derived by multiplying V by the thrust at that speed. For the military power turbojet, the thrust is nearly constant so power available is a straight line originating at V=0. The slope of the curve is directly proportional to the magnitude of thrust.

For the turboprop, the thrust tends to decrease with an increase in velocity as also seen in figure 7.6. Depending on the exact shape of the turboprop thrust curve, the maximum value of excess thrust might occur at a speed less than $V_{L/D_{max}}$ and close to the speed for minimum thrust horsepower required. Propeller efficiencies also account for part of the curve shape. The slope of the curve for power available changes as the true airspeed changes.

Altitude has an effect on rate of climb similar to its effect upon climb angle. Rate of climb at the absolute ceiling goes to zero when $T_{N_x} = D$ and excess power is minimum. The service ceiling and combat ceiling, defined as the altitudes where 100 ft/min and 500 ft/min rates of climb can be maintained, are also finite altitudes.

Weight affects rate of climb directly and in the same manner as it does climb angle. Increasing weight with no change in excess power reduces rate of climb.

Wind affects rate of climb negligibly unless gradient and direction changes are large within the air mass.

True airspeed strongly affects rate of climb performance since thrust and drag are functions of velocity, and specific excess power directly depends upon true airspeed according to Eq 7.15. Maximum rate of climb for the jet occurs at much higher true airspeed than that for L/D_{max}.

7.3.2.5 TIME TO CLIMB

The rates of climb discussed are instantaneous values. At each altitude, there is one velocity which yields maximum rate of climb. That value pertains only to the corresponding altitude. Continuous variations in rate of climb must be attained through integration as in Eq 7.17.

$$t = \int_0^h \frac{1}{\frac{dh}{dt}} dh = \int_0^h \frac{1}{ROC} dh \qquad \text{(Eq 7.17)}$$

Where:

h	Tapeline altitude	ft
ROC	Rate of climb	ft/s
t	Time	s.

The term dh/dt in Eq 7.17 is not usually available as an analytical function of altitude. The determination of time to climb then can only be determined graphically as in figure 7.7.

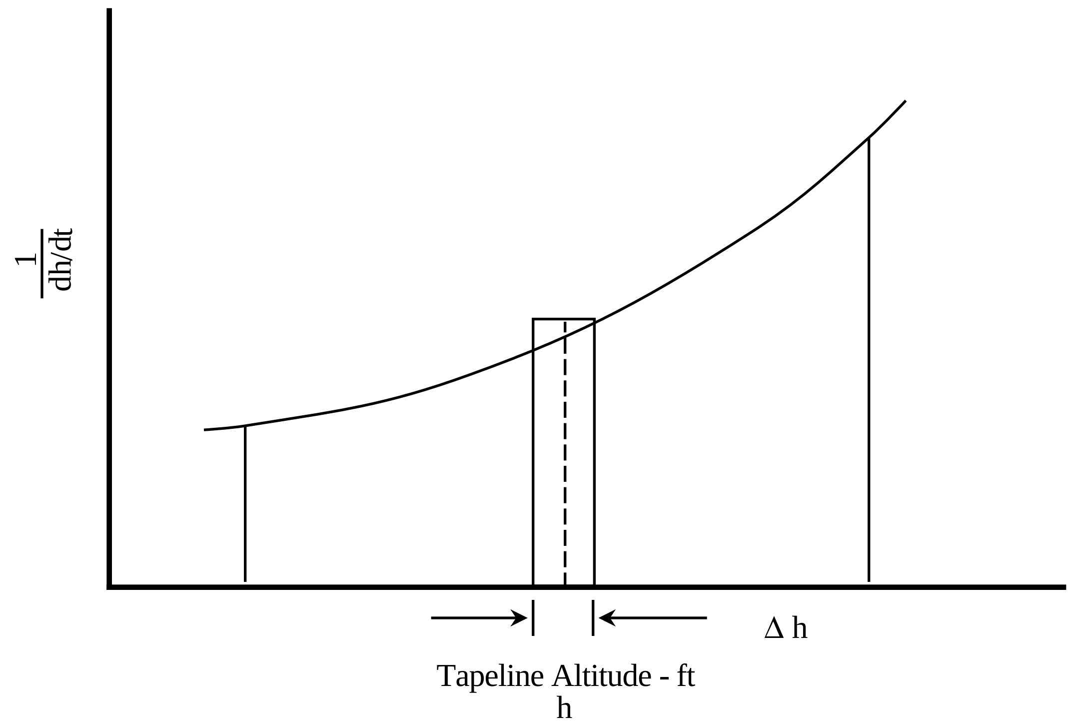

Figure 7.7
TIME TO CLIMB INTEGRATION

The method is limited by the fact that more information is needed than just the altitude change with time. Both altitude and airspeed must be specified, as well as the climb path itself and actual rate of climb at every point. A more accurate method to determine time to climb is discussed in section 7.3.3

7.3.2.6 SUMMARY OF STEADY STATE CLIMB

A curve of vertical velocity versus horizontal velocity can be used to summarize the steady state performance. A rate of climb curve is shown in figure 7.8.

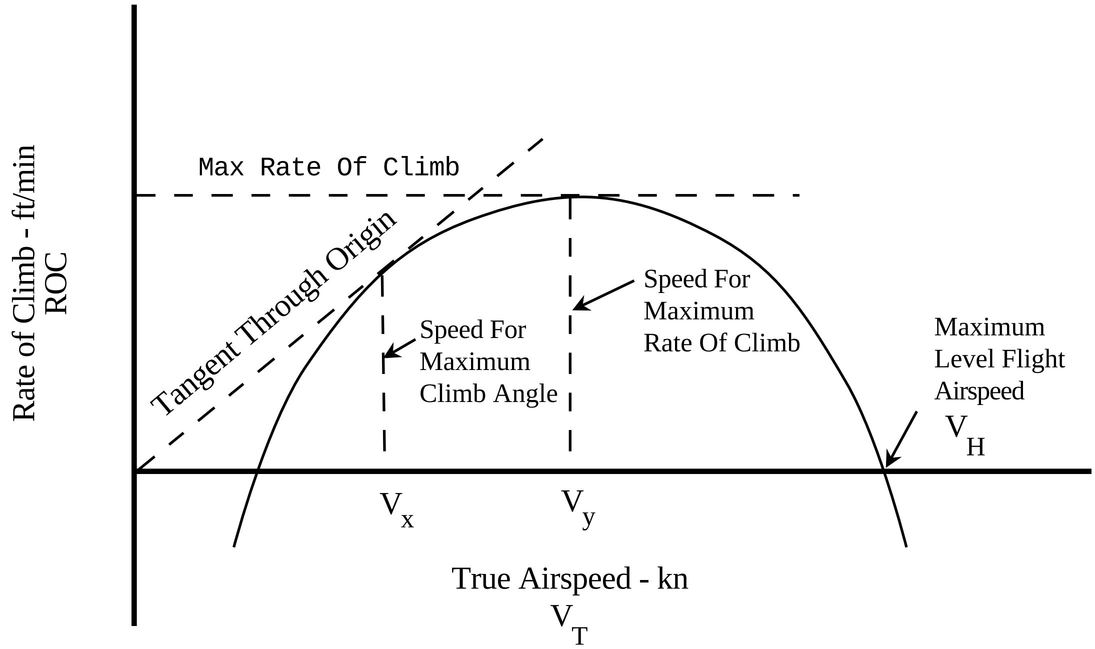

Figure 7.8
CLASSIC RATE OF CLIMB

If figure 7.8 is converted into vertical velocity versus horizontal velocity using Eqs 7.17 and 7.18, climb performance can be summarized graphically as in figure 7.9 for the same speed range.

$$V_v = V_T \sin \gamma \qquad \text{(Eq 7.18)}$$

$$V_{hor} = V_T \cos \gamma \qquad \text{(Eq 7.19)}$$

Where:

γ	Flight path angle	deg
V_{hor}	Horizontal velocity	ft/s
V_T	True airspeed	ft/s
V_v	Vertical velocity	ft/s.

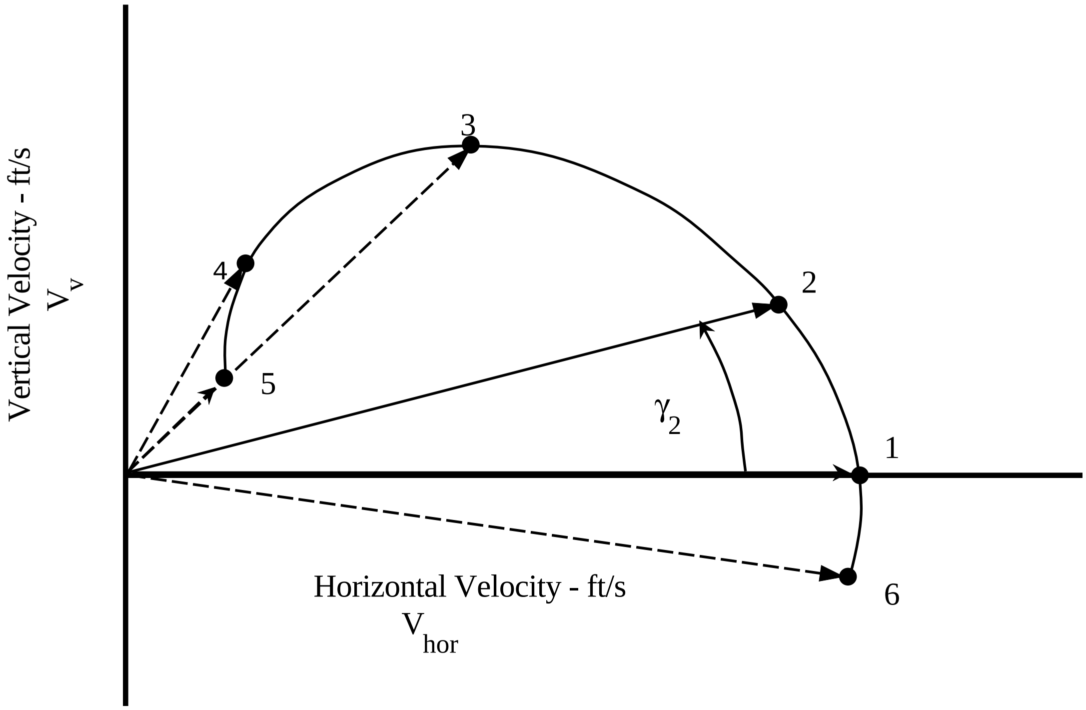

Figure 7.9
PERFORMANCE HODOGRAPH

A radius vector from the origin to any point on the plot represents the true airspeed and makes an angle to the horizontal equal to the actual climb angle at that speed. From figure 7.9:

Point 1	Maximum level flight airspeed	V_H
Point 2	Climb speed at the given rate of climb	V_T
Point 3	Speed for maximum rate of climb	V_y
Point 4	Speed for maximum climb angle	V_x
Point 5	Stall speed	V_s
Point 6	Descent	NA.

7.3.2.7 THRUST EFFECTS

The discussion to this point deals with one set of power parameters; however, rate of climb is calculated using Eq 7.16 and changes with throttle position or net thrust parallel to the flight path, T_{N_x}.

$$ROC = \frac{dh}{dt} = \frac{\left[T_{N_x} - D\right]}{W} V = \frac{T_{N_x} V - D V}{W} = \frac{P_A - P_{req}}{W} \qquad \text{(Eq 7.16)}$$

Where:

D	Drag	lb
T_{N_x}	Net thrust parallel flight path	lb
h	Tapeline altitude	ft
P_A	Power available	ft-lb/s
P_{req}	Power required	ft-lb/s
ROC	Rate of climb	ft/s
V	Velocity	ft/s
W	Weight	lb.

From a study of drag characteristics for jets, the maximum climb angle speed (V_x) occurs at $\left.\frac{C_L}{C_D}\right|_{max}$, which is the minimum drag condition. The speed is independent of power setting. The speed for best climb rate, V_y, increases with increasing thrust. Figure 7.10 illustrates the effects discussed.

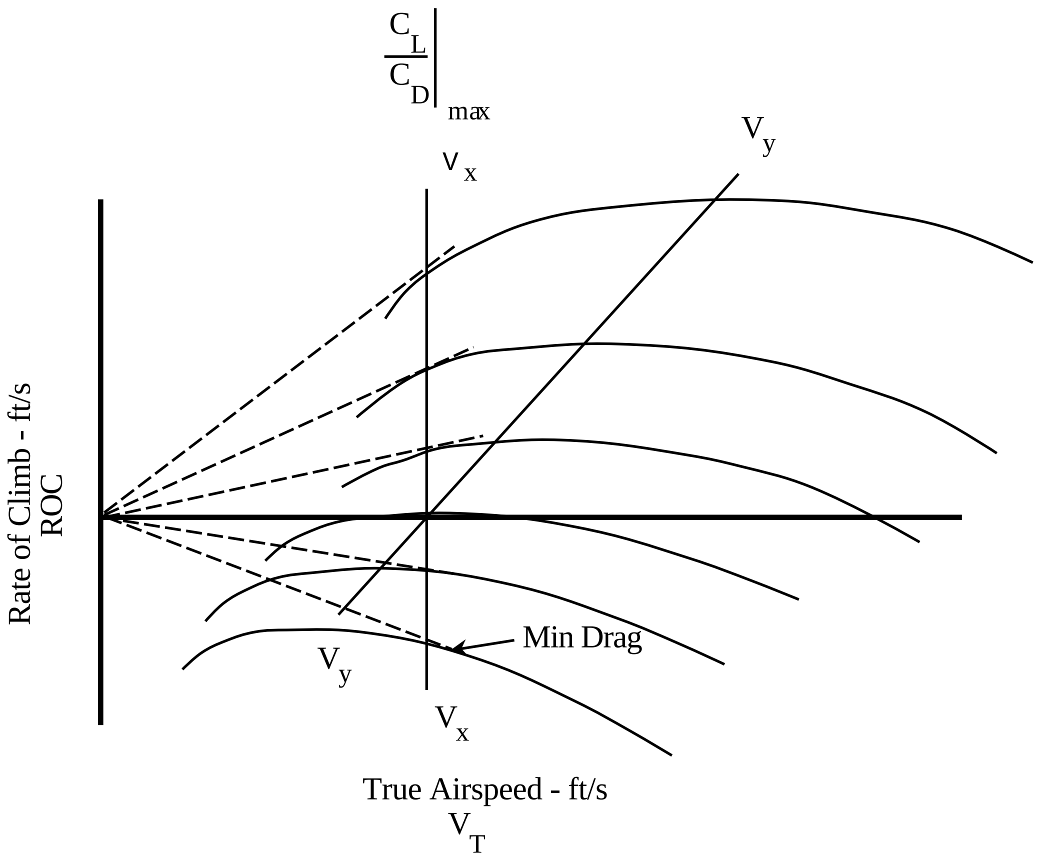

Figure 7.10
THRUST EFFECTS FOR JETS

For the propeller aircraft, the best rate of climb speed, V_y, does not change with increasing power. The speed for maximum climb angle, V_x decreases with the increased power setting as the climb progresses. Figure 7.11 shows the result.

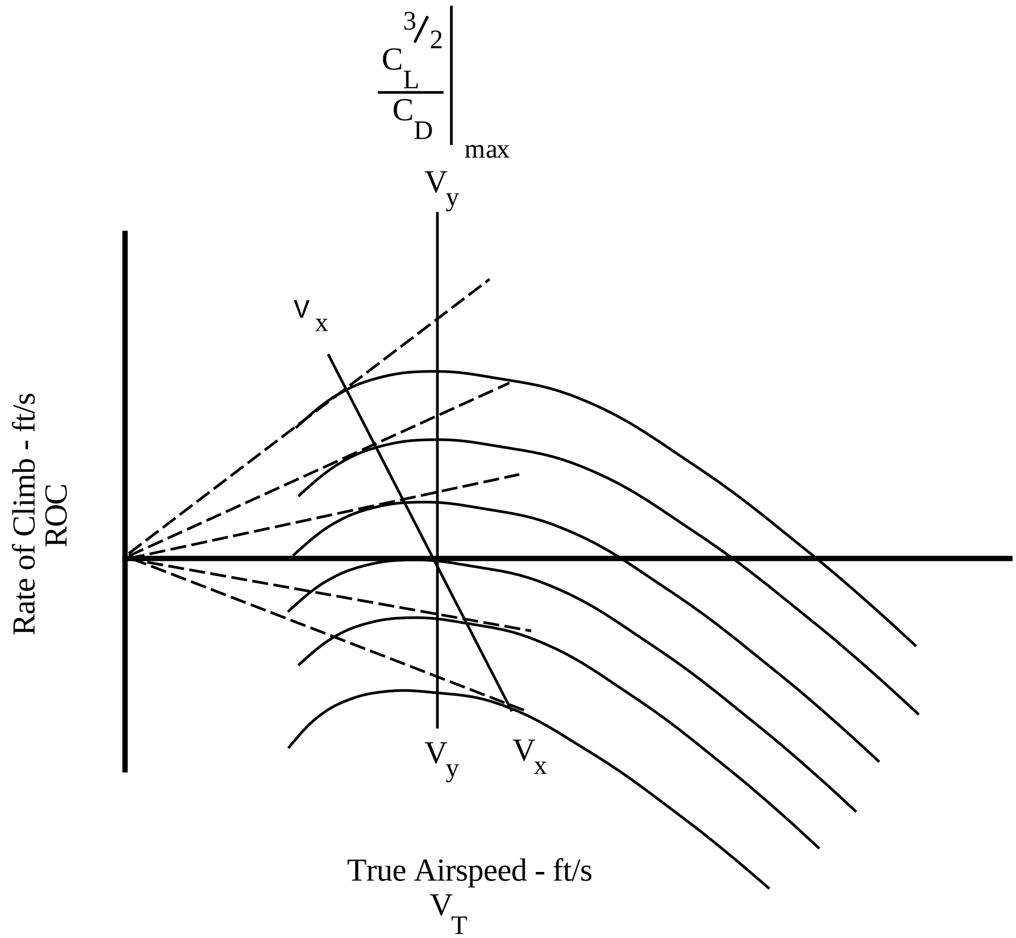

Figure 7.11

THRUST EFFECTS FOR PROPS

7.3.3 TOTAL ENERGY APPROACH TO CLIMB PERFORMANCE

Climb schedules can be determined by randomly flying a large number of climb schedules and picking the best schedule based on the flight results. Each flight requires a climb and if a schedule is selected based on measured P_S data, the results could be quite good. Schedules based on less information would yield poor results. There are more scientific approaches and this section uses the information attained from the excess power chapter to determine climb performance, and specifically climb schedules. The treatment starts with the subsonic jet and advances to supersonic speeds.

7.3.3.1 TIME BASED CLIMB SCHEDULE

P_S data taken from acceleration runs, sawtooth climbs, or other methods can be worked up on a cross-plot of altitude and Mach number, or true airspeed, with lines of constant energy height as shown in figure 7.12.

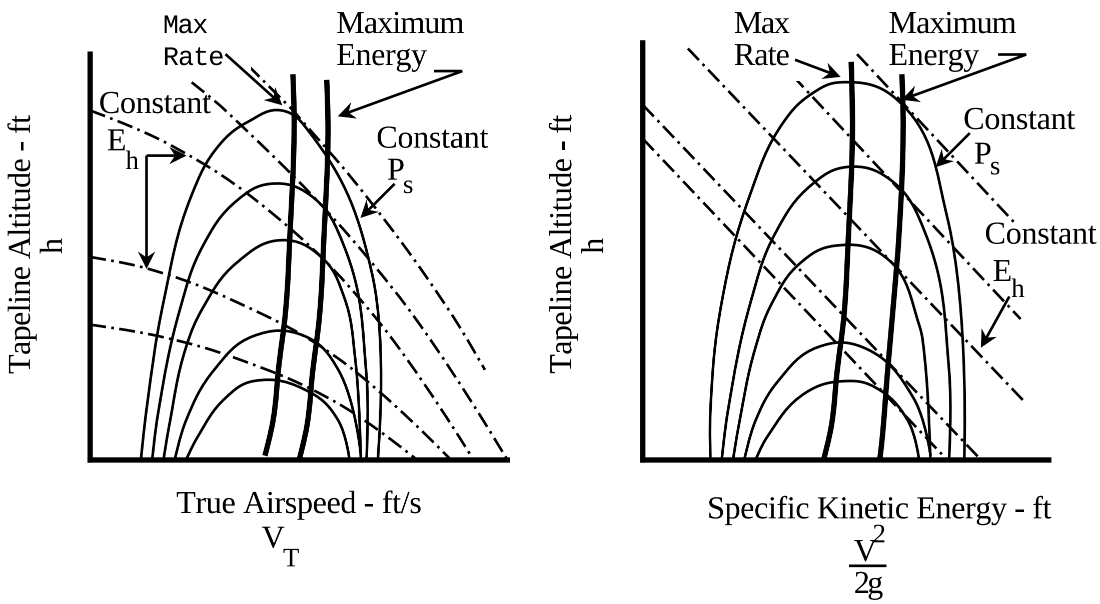

Figure 7.12
SUBSONIC CLIMB SCHEDULE

The peaks of the curves represent the speed at which the maximum specific excess power occurs at each altitude. Each peak is also the speed for maximum instantaneous rate of climb at that altitude for an aircraft flying at constant true airspeed. The maximum points occur at increasing airspeed; thus, as altitude increases, acceleration is required along the flight path. Excess power must be divided between the requirements of climbing and acceleration, which is seen as an increase in total energy as shown in Eq 7.1.

$$E_h = h + \frac{V_T^2}{2\,g} \qquad \text{(Eq 7.1)}$$

Where:

E_h	Energy height	ft
g	Gravitational acceleration	ft/s^2
h	Tapeline altitude	ft
V_T	True airspeed	ft/s.

By flying the points where the P_s contours are tangent to the lines of constant energy height, a schedule for the minimum time to achieve an energy state, or maximum rate of total energy addition, is developed as the optimum energy climb schedule.

The plot is also shown as a function of specific potential energy (h) and specific kinetic energy ($\frac{V^2}{2g}$). This curve may be useful if the points of tangency are not well defined.

7.3.3.1.1 SPECIFIC ENERGY VERSUS TOTAL ENERGY

Time to climb is given in Eq 7.17. The integration requires determining the actual rate of climb at every point, which cannot be easily obtained from figure 7.12. The integration will not work for any portion of the schedule where altitude is not increasing. To put Eq 7.17 in more useful terms the following equations are used:

$$P_s = \frac{dE_h}{dt} = \frac{dh}{dt} + \frac{V_T}{g}\frac{dV_T}{dt} \qquad \text{(Eq 7.2)}$$

$$\text{Rate of Climb} = P_s\left(\frac{1}{1 + \frac{V}{g}\frac{dV_T}{dh}}\right) \qquad \text{(Eq 7.20)}$$

$$\text{Time to Climb} = \int_{h_1}^{h_2} \frac{\left(1 + \frac{V}{g}\frac{dV_T}{dh}\right)}{P_s}\,dh \qquad \text{(Eq 7.21)}$$

$$dt = \frac{dh + \frac{V}{g}\, dV_T}{P_s} \qquad \text{(Eq 7.22)}$$

Using the differential form of Eq 7.1 and substituting it into equation 7.21, time to climb can be expressed in terms which can be graphically integrated from figure 7.12.

$$dE_h = dh + \frac{V}{g}\, dV \qquad \text{(Eq 7.23)}$$

$$dt = \frac{dE_h}{P_s} \qquad \text{(Eq 7.24)}$$

$$\text{Time to Climb} = \int_{E_{h_1}}^{E_{h_2}} \frac{1}{P_s}\, dE_h \qquad \text{(Eq 7.25)}$$

Where:

E_h	Energy height	ft
E_{h1}	Energy height at start of climb	ft
E_{h2}	Energy height at end of climb	ft
g	Gravitational acceleration	ft/s^2
h	Tapeline altitude	ft
h_1	Tapeline altitude start of climb	ft
h_2	Tapeline altitude end of climb	ft
P_s	Specific excess power	ft/s
ROC	Rate of climb	ft/s
t	Time	s
V	Velocity	ft/s
V_T	True airspeed	ft/s.

Results for both the maximum rate of climb schedule and the maximum energy climb schedule are shown in figure 7.13.

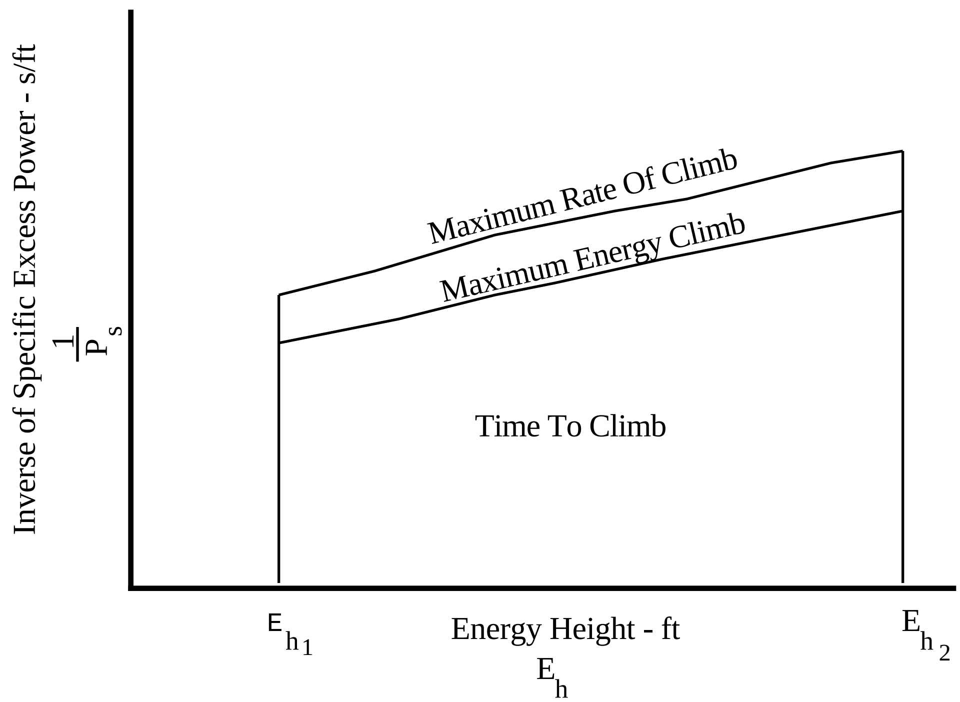

Figure 7.13
MINIMUM TIME TO CLIMB

The maximum energy climb schedule gives a path defined by altitude and Mach number for transitioning from one energy level to a higher level in the minimum time. Every point on the maximum energy climb schedule represents the maximum P_S for that energy height which will get the aircraft to an energy level faster than any other schedule. However, the potential energy is lower than in the maximum rate of climb with kinetic energy making up the difference. The maximum energy climb schedule will give the minimum time between two energy levels; however, the potential gains by flying this schedule can be negated by the real process of exchanging kinetic and potential energies. Theoretical treatments usually assume an ideal model in which the airplane can translate instantaneously, and without loss, along lines of constant energy height. The ideal model works well when the end points of the energy climb are near the maximum energy climb schedule, but breaks down when they are not, particularly if the energy levels are close together. For transitions between widely separated energy levels, the maximum energy climb schedule is nearly optimal and is recommended for jets climbing to high altitude. The climb schedule actually recommended is often a compromise between the theoretical maximum rate and maximum energy schedules, and may be further modified by considerations such as providing an airspeed or Mach number profile which is easy to fly, and/or providing a Mach number relative to maximum range or maximum endurance airspeed.

For a supersonic aircraft, the energy schedule becomes more significant. Figure 7.14 illustrates a typical climb schedule for a supersonic aircraft.

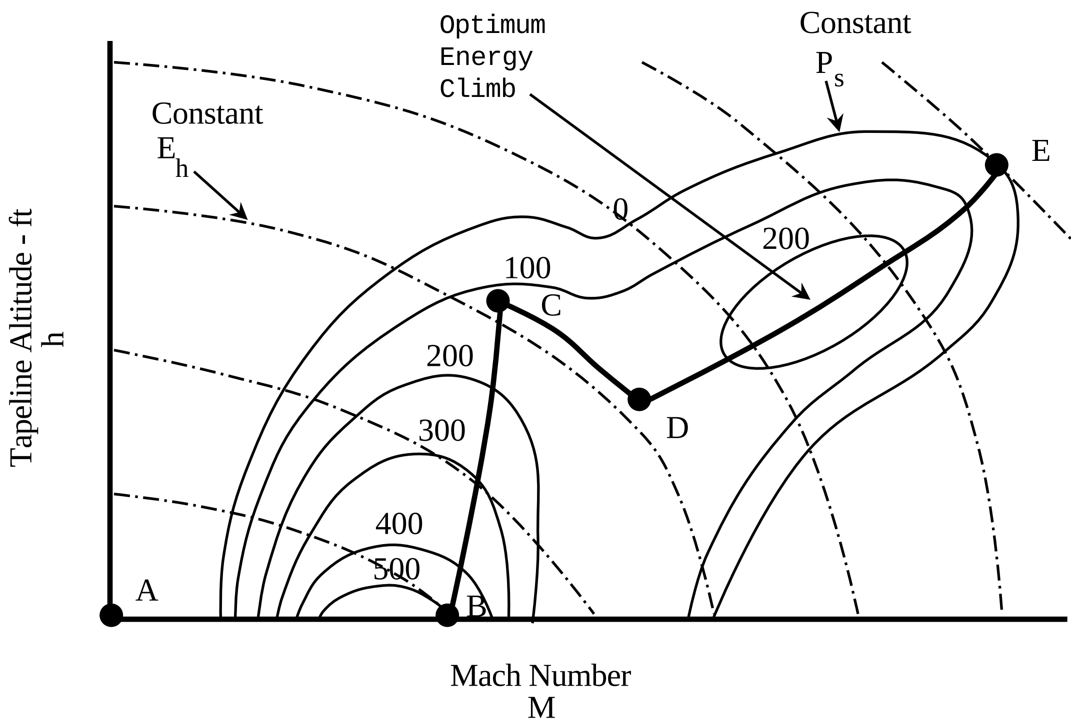

Figure 7.14
SUPERSONIC CLIMB SCHEDULE

If the final speed is near the aircraft's maximum speed, the large speed increase necessary renders the conventional method of using the peaks of the P_s curves useless. However, the energy method works well. Note in this example the optimum climb path includes an acceleration in a dive. This optimum energy climb path is also known as the Rutowski climb path, after its developer. The path (Figure 7.14) consists of four segments to reach energy state E in minimum time. Segment AB represents a constant altitude acceleration from V = 0 to climb speed at state B. The subsonic climb segment follows a path similar to the one illustrated to the tropopause at state C. This subsonic climb is usually a nearly constant Mach number schedule. An ideal pushover or dive is carried out at constant E_h from C to D. The acceleration in the dive is actually part of the optimum climb path. Segment DE is the supersonic climb to the final energy state desired at E. Note segments BC and DE pass through points on P_s contours which are tangent to lines of constant E_h. A climb schedule defined by the conventional method of the peaks of the P_s curves at each altitude is undesirable because of the large speed change involved if a speed near the maximum is desired at the end of the climb. However, the conventional schedule may still be useful if a profile is desired to reach maximum range airspeed at altitude.

When and how to transition from the subsonic segment to the supersonic segment may be an issue if the P_s contours near M = 1 are poorly defined. There is no complete agreement on when to start the pushover. Perhaps the most expeditious path is the one toward the highest P_s contour available without decreasing E_h. The assumption implies the climb should be subsonic until intercepting an E_h level tangent to two P_s contours of equal value, one in a subsonic region and the other in the supersonic region. Path CD in figure 7.14 illustrates a typical transition using this idea. Figure 7.15 illustrates the difficulty in choosing the transition paths when P_s contours become irregular in the transonic region.

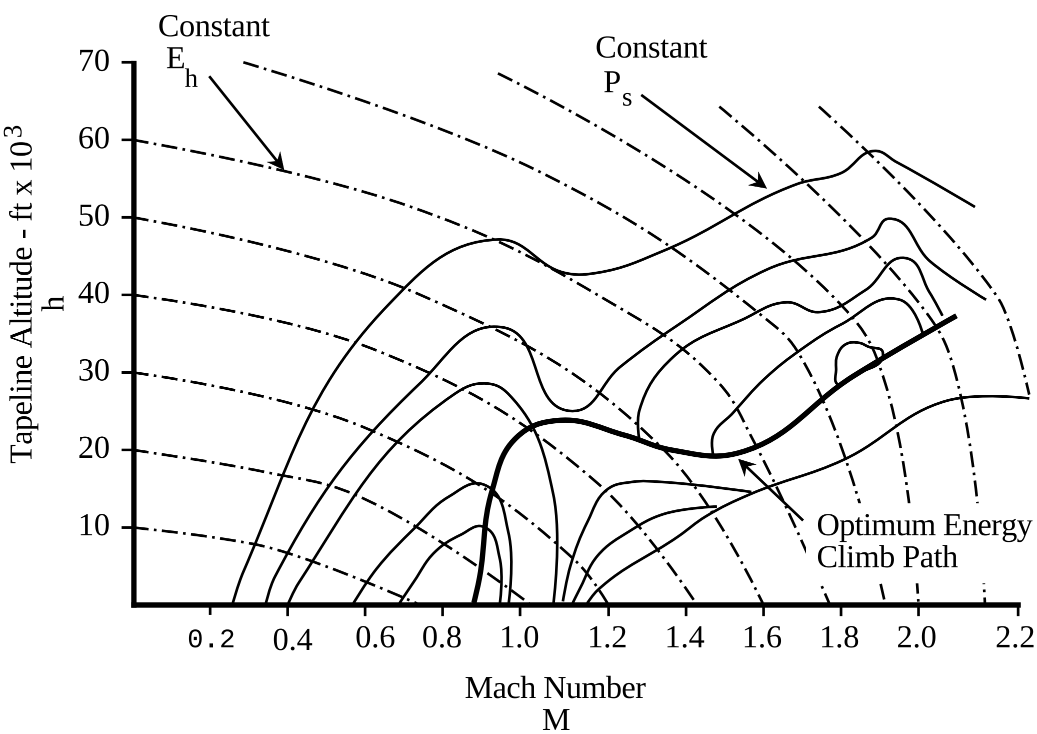

Figure 7.15
TYPICAL ENERGY LEVEL CLIMB PATH

Notice in figure 7.15, real transitions as opposed to ideal zooms and dives, where transitions are assumed to be instantaneous, result in a climb path with the corners rounded off. The abrupt discontinuities in angle of attack and attitude are avoided in the actual climb.

7.3.3.2 FUEL BASED CLIMB SCHEDULE

The aircraft mission may require the expenditure of minimum fuel to achieve a given energy level. The energy approach to climb performance can also be used to determine how much total energy is added per pound of fuel consumed. This requires specific energy to be differentiated with respect to the change in aircraft gross weight due to fuel used. Change in altitude per pound of fuel used is dh/dW in climbs, and the change in airspeed per pound of fuel in accelerations is dV/dW. The sum of both terms equals the change in specific energy with respect to fuel used as in Eq 7.26.

$$\frac{dE_h}{dW} = \frac{d}{dW}\left(h + \frac{V_T^2}{2\,g}\right) = \frac{dh}{dW} + \frac{V_T}{g}\,\frac{dV_T}{dW} \qquad \text{(Eq 7.26)}$$

Fuel burned in a climb can be determined by integrating Eq 7.26 as was done in determining time to climb.

$$\text{Fuel to Climb} = \int_{W_1}^{W_2} dW = -\int_{E_{h_1}}^{E_{h_2}} \frac{1}{\frac{dE_h}{dW}}\, dE_h \qquad \text{(Eq 7.27)}$$

A relationship of fuel flow to specific excess power can be written as Eq 7.28.

$$-\frac{1}{\frac{dE_h}{dW}} = -\frac{dW}{dE_h} = -\frac{dW}{dt}\,\frac{dt}{dE_h} = \frac{\dot{W}_f}{P_s} \qquad \text{(Eq 7.28)}$$

Eq 7.27 can be rewritten as follows:

$$\text{Fuel to Climb} = \int_{E_{h_1}}^{E_{h_2}} \frac{\dot{W}_f}{P_s}\, dE_h = \int_{E_{h_1}}^{E_{h_2}} \frac{1}{\frac{P_s}{\dot{W}_f}}\, dE_h \qquad \text{(Eq 7.29)}$$

Where:

E_h	Energy height	ft
g	Gravitational acceleration	ft/s^2
h	Tapeline altitude	ft
P_s	Specific excess power	ft/s
t	Time	s
V	Velocity	ft/s
V_T	True airspeed	kn
W	Weight	lb
$\dot{W}_f$	Fuel flow	lb/h.

Evaluating Eq 7.29 requires combining P_s and fuel flow data from other tests. From acceleration run data, the P_s curve similar to the solid line in figure 7.16, and referred fuel flow curve, similar to figure 7.17, can be generated.

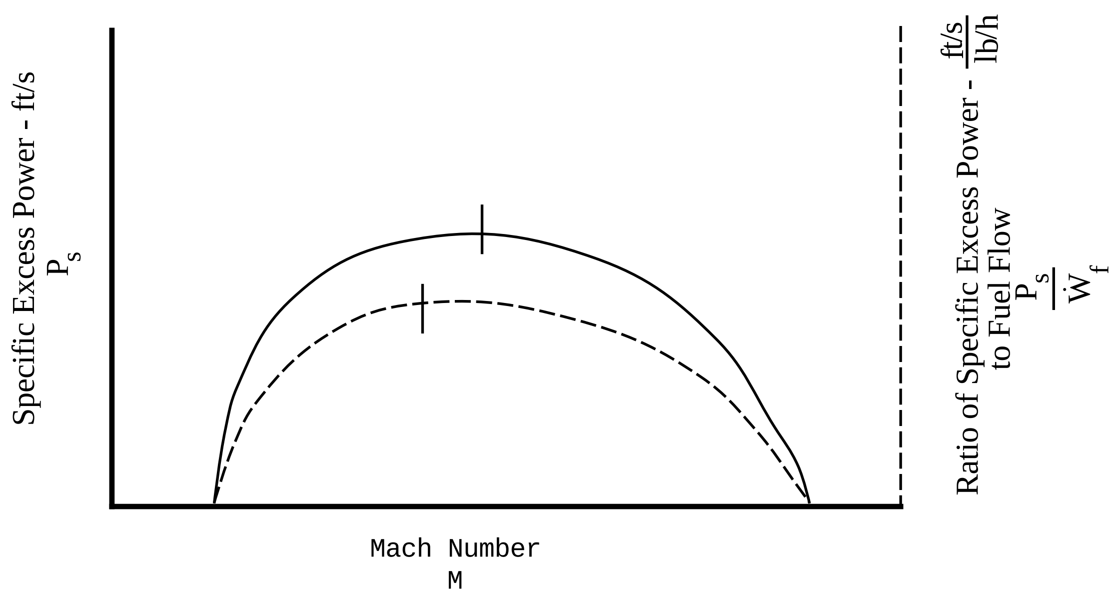

Figure 7.16

SPECIFIC EXCESS POWER CURVE

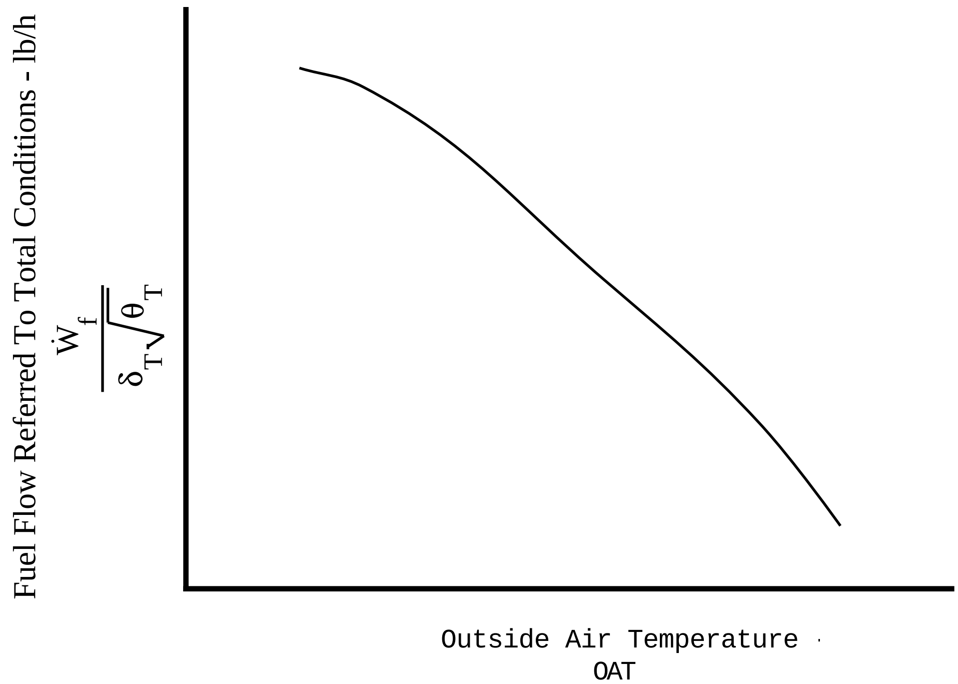

Figure 7.17
REFERRED FUEL FLOW

For a given altitude (δ) and temperature (θ), the outside air temperature (OAT) and referred fuel flow can be determined versus Mach number. For the same acceleration run, the actual fuel flow for standard conditions can be determined as shown in figure 7.18.

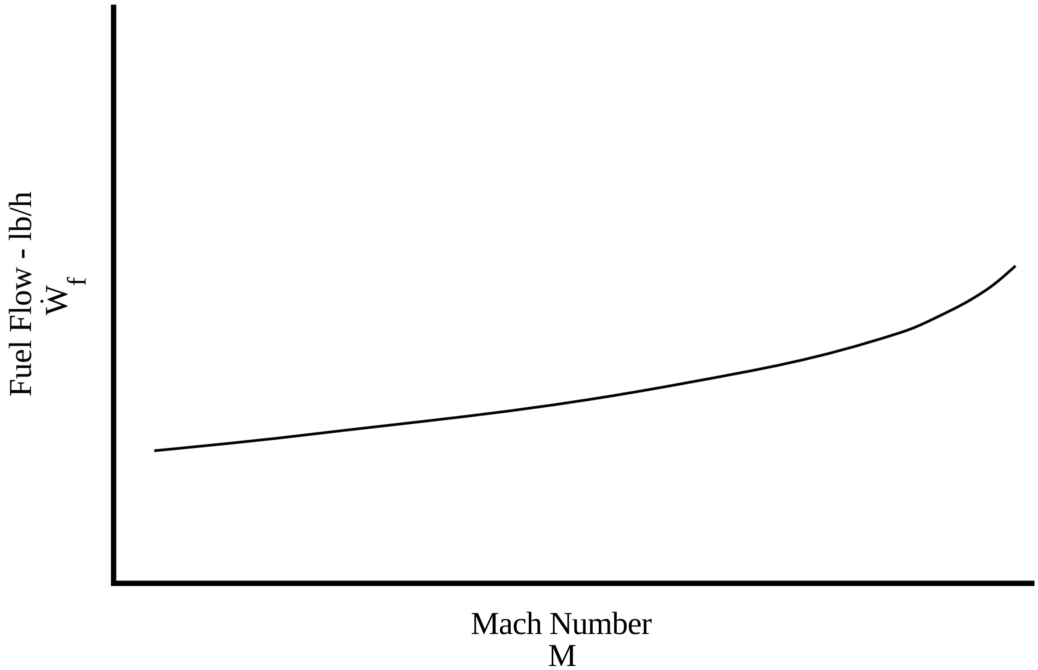

Figure 7.18
ACTUAL FUEL FLOW

The ratio of P_s to $\dot{W}_f$ can be determined by dividing each point on figure 7.16 by the corresponding points on figure 7.18 and plotting $\frac{P_s}{\dot{W}_f}$ versus Mach number as the dashed curve in figure 7.16. Since fuel flow increases with increasing speed, the peak of the $\frac{P_s}{\dot{W}_f}$ curve occurs at a slower speed than the peak of the P_s curve in figure 7.16.

To develop the minimum fuel climb schedule, all the $\frac{P_s}{\dot{W}_f}$ curves for each altitude are determined. The results are cross-plotted as shown in figure 7.19.

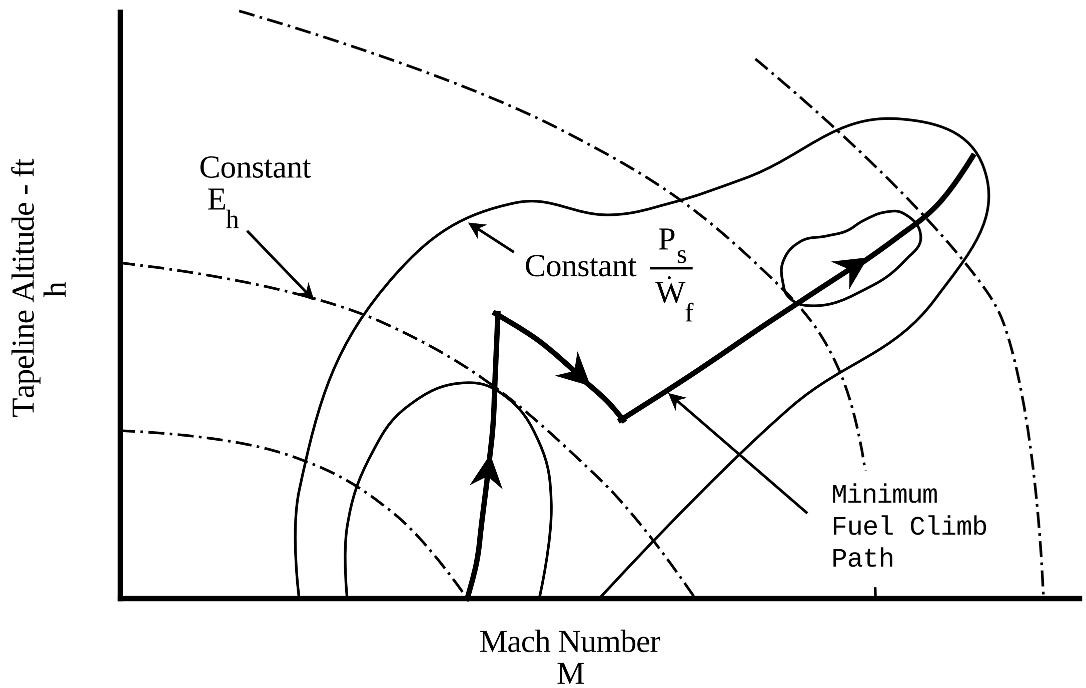

Figure 7.19
MINIMUM FUEL CLIMB SCHEDULE

7.3.3.2.1 TIME VERSUS FUEL BASED CLIMB

The climb path based on minimum fuel in figure 7.19 appears very much like the time based climb in figure 7.14. Figure 7.20 depicts a typical result given the same aircraft following an optimal time climb and an optimal fuel climb.

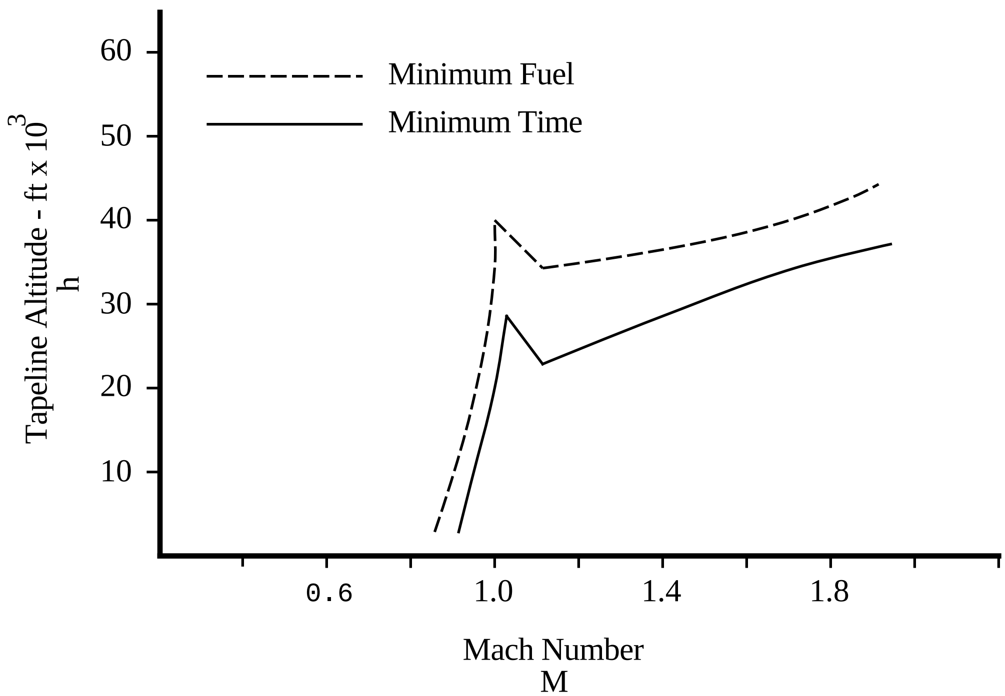

Figure 7.20
OPTIMAL TIME AND FUEL CLIMBS

The optimal minimum fuel climb path lies above, but roughly parallel, the optimal minimum time climb path. In reality, the optimal fuel path is easier to achieve since the optimal time case requires an ideal climb, an ideal dive, and an ideal zoom to reach the end point.

7.3.3.2.2 MAXIMUM RANGE CLIMB SCHEDULE

The maximum range climb schedule should achieve maximum range for fuel used during the climb to a desired altitude and cruise speed. There are sophisticated mathematical techniques which can be used to determine a theoretical solution. In practice, if the cruise speed is near the climb speed for an energy minimum fuel climb, the minimum fuel climb approximates the no-wind maximum range per pound of fuel.

The initial point of the cruise leg is defined by an altitude and an airspeed for a particular gross weight. The schedule which gets to that energy level with minimum fuel used, is close to the optimal time schedule. However, actual down range distance traveled in the climb is considered when optimizing the overall range problem which also includes cruise and descent.

As in the cruise phase, wind affects the range in a climb. For a head wind, an incremental increase in speed for the climb schedule slightly improves the range characteristic provided the peaks of the $\frac{P_s}{\dot{W}_f}$ curves, with respect to energy height, are not too sharp. For a tail wind, an incremental decrease in speed increases range over that obtained from the no-wind schedule.

7.4 TEST METHODS AND TECHNIQUES

7.4.1 SAWTOOTH CLIMB

A series of timed climbs are made at different speeds from a point below the test altitude to a point above it. Speeds are chosen to bracket the expected best climb speed of the aircraft. Power setting is defined by the scope of the test but is the same for each run. Normally, full power is used. A typical flight data card is shown in figure 7.21.

Target V_o	V_o	Initial H_P	Final H_P	Δt	Fuel	OAT	Misc

Figure 7.21
SAWTOOTH DATA CARD

Climbs are performed at the same power setting and aircraft configuration as used in the check climb (paragraph 7.4.3). The same altitude band is used for each climb, until P_s decreases and time, rather than altitude gain, becomes the test criterion. The altitude increment is about 1000 ft either side of a target altitude, or a height change attainable in two minutes, which ever is less. When time is the criterion, choose the climb band symmetrically about the target altitude.

The aircraft is first trimmed in the configuration desired while still well below the nominal altitude. Power is applied and final trim adjustments are made before reaching the lower limit of the altitude band being measured.

The exact time of entering and leaving the altitude band is recorded by a stopwatch or an instrumentation system.

Once through the altitude increment, data are recorded, and a descending turn is initialized to get the aircraft below the altitude band for another run. As many points as possible are flown at each altitude. In addition, a full power unaccelerated minimum speed point, and a maximum speed point are obtained at the test altitude in order to complete the curve.

Climb perpendicular to the prevailing winds to minimize the effects of wind shear. Confine the flights to the bounds of a limited geographical area since the primary concern is the shape of the curve obtained rather than the magnitude. For each altitude, a standard data card is prepared similar to figure 7.21 with the target indicated airspeed included for each point.

Record actual in-flight V_o, W, time, fuel counts or fuel remaining, and either outside air temperature or time of day so temperature can be obtained by other meteorological methods. The card can be expanded to record other parameters such as angle of attack, engine RPM, torque, etc. On the back of the data card, keep a running plot of observed time to climb versus V_o and before leaving the test altitude, examine the plot for points which need repeating.

The description above is only applicable when P_s is determined to be positive. If P_s is negative and the airplane is descending, then the same flight test technique applies, except in reverse. For a description of the sawtooth descent flight test technique, see paragraph 8.4.1.

7.4.1.1 DATA REQUIRED

1. Time: Record elapsed time from the beginning of the altitude band to the end, or two minutes whichever comes first.
2. Altitude: Record the observed pressure altitude H_{P_o} band for each point.

3. Velocity: Record observed airspeed, V_o.

4. OAT or T_a: Record ambient air temperature from on-board instrumentation at target altitude. (May be obtained from direct observation).

5. Fuel weight: Record the fuel remaining to determine aircraft gross weight.

6. Miscellaneous: Record other information desired such as RPM, angle of attack, and torque for a turboprop.

7.4.1.2 TEST CRITERIA

Allow sufficient altitude for the engine(s) to reach normal operating temperatures and the airplane to be completely stabilized at the desired airspeed before entering the data band. Smoothness is just as important as in acceleration runs and for the same reasons. If a small airspeed error is made while setting the airplane up, it is better to maintain the incorrect speed as accurately as possible, rather than try to correct it and risk aborting the entire run.

7.4.1.3 DATA REQUIREMENTS

1. Test altitude band ± 1000 ft about a target altitude.
2. V_o ± 1 kn with smooth corrections.
3. Normal acceleration ± 0.1 g.

7.4.1.4 SAFETY CONSIDERATIONS

There are no unique hazards or safety precautions associated with sawtooth climbs. Observe airspeed limits when in the powered approach configuration and engine limits at the selected thrust setting. Consider low altitude stall potential in slow speed climb tests and V_{mc} for multi-engine aircraft.

7.4.2 CHECK CLIMB TEST

The check climb test is flown to compare the standard day climb performance of an aircraft in a specific configuration to results predicted from sawtooth climbs or acceleration runs. The three main areas of investigation are:

1. Time to climb.
2. Distance traveled.
3. Fuel used.

In addition, data may be obtained on various engine parameters such as engine speed, exhaust gas temperature, engine pressure ratio, gross thrust, angle of attack, etc. These are useful to the analyst but are secondary to the three main parameters. The general method is to climb the aircraft to just below the maximum ceiling while maintaining precisely a predetermined climb schedule. This schedule may be a best climb schedule as obtained by flight test, a schedule recommended by the manufacturer, or some other schedule for which climb performance is of interest. Specify the schedule flown on each climb performance chart.

Record data at approximately equal increments of altitude and include time, speed, fuel flow, temperature, and any other desired parameters. For most jet aircraft, a mechanical recording means is necessary to obtain simultaneous reading of the many parameters of interest.

After the schedule to be flown is determined, data cards are prepared to record the climb data as in figure 7.22.

	t	Fuel temp	Fuel density	OAT or T_a	Fuel remaining	
Prior to Start	NA					
Start		NA	NA			
Taxi		NA	NA			
Takeoff		NA	NA			
t	H_{P_o}	V_o target	V_o actual	OAT or T_a	Fuel remaining	Misc

Figure 7.22
CHECK CLIMB

Adjust target points for instrument error and position error for both the airspeed indicator and altimeter. If the anticipated rate of climb is low, data are recorded every 1000 to 2000 ft. If the rate of climb is high, every 5000 ft is sufficient. The interval is adjusted as the rate of climb decreases with altitude.

An area of smooth air, light winds, and stable temperature gradients from ground level to the aircraft's maximum ceiling is desirable. The test area can be sampled via a survey balloon or another aircraft for wind and temperature data. Plan the flight to climb perpendicular to the wind direction.

Since gross weight, fuel weight, and fuel density are extremely important to climb tests, fuel and weigh the aircraft prior to commencement of tests. Fuel samples are taken and tested for temperature and density. Record fuel remaining and time for start, taxi, takeoff, and acceleration to climb schedule whenever conditions permit.

Establish level flight as low as possible on climb heading. If the rate of climb is high, the best entry is achieved by first stabilizing in level flight with partial power at some speed below the scheduled climb speed. Trim the aircraft for hands-off flight. When all preparations are complete and the data recorder is running, apply power, and as the climb speed is approached, rotate to intercept and maintain the climb schedule.

If rate of climb is fairly low, a better entry can be achieved by stabilizing on the target speed 1000 ft below entry altitude. When preparations are complete and the aircraft is trimmed, advance the power smoothly, and rotate the aircraft simultaneously to maintain airspeed. As the desired power setting is reached, stop the rotation, at which time the aircraft is approximately established on the climb schedule.

During the climb, trim the aircraft. Maintain the climb schedule to within 5 kn, if possible. A rapid cross-check between external horizon and the airspeed indicator is required. If the pitch attitude is very steep, it may be necessary to substitute the aircraft attitude indicator for the external horizon during initial portions of the climb.

Wind gradients appear as sudden airspeed changes. If these affect the climb speed schedule, the appropriate corrective action is a small, but immediate attitude correction. If the wind gradient effect subsides, apply an appropriate corrective action.

At high altitudes, maintaining a precise speed schedule becomes difficult. A slight rate of change of indicated airspeed implies a much larger rate of change of kinetic energy. Any undesirable trend is difficult to stop since the aerodynamic controls are less effective. The best way to cope is to avoid large corrections by a rapid cross-check, precise control, and constant attention to trim. If corrections become necessary, avoid over controlling due to the hysteresis in the airspeed indicator.

If the climb must be interrupted, stop the climb at a given pressure altitude, noting V_o, fuel, time, and distance if available. Descend below the altitude at which the climb was stopped. Maneuver as required and re-intercept the climb schedule as soon as possible to minimize gross weight change. Intercept the climb at the break off pressure altitude and airspeed after re-establishing attitude and stabilizing the climb.

The test for a turboprop aircraft is identical to the jet test. In this case however, engine torque and engine shaft horsepower (ESHP) are adjusted in the climb. The engine controls have to be managed to ensure optimum climb power is maintained.

7.4.2.1 DATA REQUIRED

Record the following data at each climb increment which should be as often as possible but no greater than each 5000 ft or 2 minutes for hand held data:

1. Time (t).
2. Observed airspeed (V_o).
3. Observed pressure altitude (H_{P_o}).
4. Temperature (OAT or T_a).
5. Fuel remaining (W_f), or start and end fuel weight.
6. Distance (d).
7. Miscellaneous as desired.

7.4.2.2 TEST CRITERIA

1. Maintain coordinated wings level flight.
2. Keep turns to a minimum and use less than 10 degrees bank to keep n_z near 1.0.
3. No more than 30 degrees heading change.

7.4.2.3 DATA REQUIREMENTS

1. Airspeed ± 5 kn or 0.01 M.
2. n_z ± 0.1 g.

7.5 DATA REDUCTION

7.5.1 SAWTOOTH CLIMB

The sawtooth climb data is reduced similar to the acceleration runs, but there are some additional correction factors necessitated by this test method.

Eq 7.20 expresses test day rate of climb in terms of P_s (which is independent of the climb path) and dV_T/dh (which defines the climb path).

$$\text{Rate of Climb} = P_s \left(\frac{1}{1 + \frac{V}{g} \frac{dV_T}{dh}} \right) \qquad \text{(Eq 7.20)}$$

Recall:

$$P_s = \frac{dE_h}{dt} = \frac{dh}{dt} + \frac{V_T}{g} \frac{dV_T}{dt} \qquad \text{(Eq 7.2)}$$

E_h for the end points of each sawtooth climb segment are determined from:

$$E_{h_{Test}} = h_{Test} + \frac{V_{T_{Test}}^2}{2\,g} \qquad \text{(Eq 7.30)}$$

The slope of E_h versus time as the climb passes through the reference altitude ($H_{P_{ref}}$) is $P_{s(Test)}$. In practice, since a small altitude band is chosen, the average slope is defined by E_h(end) - E_h(start) divided by elapsed time. In a similar way, dV_T/dh is computed from V_T(end) - V_T(start) divided by altitude change. True airspeed is obtained at the reference altitude ($V_{T_{Ref}}$) by linear interpolation between the end points. Substituting these values into Eq 7.20 yields:

$$\left(\frac{dh}{dt} \right)_{Test} = P_{s_{Test}} \left(\frac{1}{1 + \frac{V_{T_{ref}}}{g} \frac{dV_T}{dh}} \right) \qquad \text{(Eq 7.31)}$$

Where:

E_h	Energy height	ft
$E_{h_{Test}}$	Test energy height	ft
g	Gravitational acceleration	ft/s^2
h	Tapeline altitude	ft
h_{Test}	Test tapeline altitude	ft
P_s	Specific excess power	ft/s

$P_{s_{Test}}$	Test specific excess power	ft/s
t	Time	s
V	Velocity	ft/s
V_T	True airspeed	ft/s
$V_{T_{ref}}$	Reference true airspeed	ft/s
$V_{T_{Test}}$	Test true airspeed	ft/s.

7.5.1.1 SPECIFIC EXCESS POWER CORRECTION

$P_{s_{Test}}$ is next corrected to standard conditions, standard weight and temperature at $H_{P\ ref}$. The correction is carried out exactly as it was for the level acceleration data with the exception of the correction for the change in induced drag:

$$P_{s_{Std}} = P_{s_{Test}} \frac{W_{Test}}{W_{Std}} \frac{V_{T_{Std}}}{V_{T_{Test}}} + \frac{V_{T_{Std}}}{W_{Std}} \left(\Delta T_{N_x} - \Delta D\right) \qquad \text{(Eq 7.32)}$$

Where:

ΔD	Standard drag minus test drag	lb
ΔT_{N_x}	Standard net thrust parallel flight path minus test net thrust	lb
$P_{s_{Std}}$	Standard specific excess energy	ft/s
$P_{s_{Test}}$	Test specific excess energy	ft/s
$V_{T_{Std}}$	Standard true airspeed	ft/s
$V_{T_{Test}}$	Test true airspeed	ft/s
W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb.

7.5.1.2 DRAG CORRECTION

The drag correction used in the level acceleration data reduction is as follows:

$$\Delta D = D_{Std} - D_{Test} = \frac{2\left(W_{Std}^2 - W_{Test}^2\right)}{\pi\, e\, AR\, S\, \gamma\, P_a\, M^2} \qquad \text{(Eq 7.33)}$$

The following assumptions are made:

1. $L = W$.
2. $T \sin \alpha_j = 0$ (no thrust lift).
3. $n_z = 1$ (level flight).

Figure 7.2 shows in a climb, even if the thrust vector is aligned with the flight path ($T_{N_x} \sin \alpha_j = 0$) the lift is not equal to the weight.

Summing the vertical forces:

$$L - W \cos \gamma + T_G \sin \alpha_j = \frac{W}{g} a_z \qquad \text{(Eq 7.8)}$$

For $T_G \sin \alpha_j = 0$, and zero acceleration:

$$L = W \cos \gamma \qquad \text{(Eq 7.10)}$$

Also, since:

$$L = n_z W \qquad \text{(Eq 7.34)}$$

For straight flight, the normal load factor is the cosine of the climb angle:

$$n_z = \cos \gamma \qquad \text{(Eq 7.35)}$$

The drag correction for a climb can now be written, assuming a parabolic drag polar, as:

$$\Delta D = \frac{2 \left(W_{Std}^2 \cos^2 \gamma_{Std} - W_{Test}^2 \cos^2 \gamma_{Test} \right)}{\pi \, e \, AR \, \rho_{ssl} \, V_e^2 \, S} \qquad \text{(Eq 7.36)}$$

Or:

$$\Delta D = \frac{2\left(W_{Std}^2 n_{z\,Std}^2 - W_{Test}^2 n_{z\,Test}^2\right)}{\pi\, e\, AR\, \rho_{ssl}\, V_e^2\, S} \qquad \text{(Eq 7.37)}$$

In applying either Eq 7.36 or Eq 7.37, the requirement is to find the drag correction in order to compute the corrected rate of climb. However, in applying Eq 7.36, the corrected rate of climb is implicit in the γ_{Std} (standard flight path angle) term of the equation. In other words, the standard rate of climb is a function of itself. Therefore, an iterative method is necessary to solve Eq 7.36.

For the first approximation set $\gamma_{Std} = \gamma_{Test}$, where γ_{Test} is computed from the test results, and calculate the induced drag correction from Eq 7.36.

The induced drag correction obtained is substituted in Eq 7.32 to yield the first iteration of $P_{s_{Std}}$, which is in turn substituted in Eq 7.38 to give the first iteration value of rate of climb and the first iteration of standard day climb angle, γ_{Std} in Eq 7.39:

$$\left(\frac{dh}{dt}\right)_{Std} = P_{s_{Std}}\left(\frac{1}{1 + \frac{V_T}{g}\frac{dV_T}{dh}}\right) \qquad \text{(Eq 7.38)}$$

$$\gamma_{Std} = \sin^{-1}\left(\frac{\frac{dh}{dt}}{V_T}\right) \qquad \text{(Eq 7.39)}$$

Where:

α_j	Thrust angle	deg
AR	Aspect ratio	
a_z	Acceleration perpendicular to flight path	ft/s^2
D	Drag	lb
D_{Std}	Standard drag	lb
D_{Test}	Test drag	lb
e	Oswald’s efficiency factor	
γ	Flight path angle	deg
	Ratio of specific heats	
g	Gravitational acceleration	ft/s^2

γ_{Std}	Standard flight path angle	deg
γ_{Test}	Test flight path angle	deg
L	Lift	lb
M	Mach number	
n_z	Normal acceleration	g
$n_{z_{Std}}$	Standard normal acceleration	g
$n_{z_{Test}}$	Test normal acceleration	g
π	Constant	
P_a	Ambient pressure	psf
ρ_{ssl}	Standard sea level air density	0.0023769 slug/ft^3
S	Wing area	ft^2
T_G	Gross thrust	lb
V_e	Equivalent airspeed	ft/s
V_T	True airspeed	ft/s
W	Weight	lb
W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb.

This value of γ then becomes γ_{Std} in the induced drag correction and the iteration is repeated until γ_{Std} is no longer changing. For airplanes with modest climb capabilities, γ is small and the iteration closes quickly. For steep climb angles the situation is different and as γ approaches 90˚ the iteration may become unstable. However, this situation is not likely to occur under the conditions in which sawtooth climbs test techniques are chosen.

7.5.1.3 THRUST LIFT CORRECTION

All the previous corrections assumed no contribution to lift from the inclination of the thrust line to the flight path. This condition is not generally true, though the errors introduced by this assumption are small enough to be neglected for most cases. However, at low speed and high angle of attack, thrust lift must be taken into consideration, as shown in figure 7.2.

Eq 7.8 yields:

$$L = W \cos\gamma - T_G \sin\alpha_j \qquad \text{(Eq 7.40)}$$

Since induced drag depends on normal acceleration:

$$n_z = \frac{L}{W} \qquad \text{(Eq 7.41)}$$

$$n_z = \cos\gamma - \frac{T_G}{W}\sin\alpha_j \qquad \text{(Eq 7.42)}$$

The equation for the sum of the forces in the horizontal direction is affected since:

$$T_{N_x} = T_G \cos\alpha_j - T_R \qquad \text{(Eq 7.12)}$$

In Eq 7.32, both the ΔT_{N_x} and ΔD terms are affected by thrust lift.

$$P_{s_{Std}} = P_{s_{Test}} \frac{W_{Test}}{W_{Std}} \frac{V_{T_{Std}}}{V_{T_{Test}}} + \frac{V_{T_{Std}}}{W_{Std}} (\Delta T_{N_x} - \Delta D) \qquad \text{(Eq 7.32)}$$

Where:

α_j	Thrust angle	deg
ΔD	Standard drag minus test drag	lb
ΔT_{N_x}	Standard net thrust parallel flight path minus test net thrust	lb
γ	Flight path angle	deg
L	Lift	lb
n_z	Normal acceleration	g
$P_{s_{Std}}$	Standard specific excess power	ft/s
$P_{s_{Test}}$	Test specific excess power	ft/s
T_G	Gross thrust	lb
T_{N_x}	Net thrust parallel flight path	lb
T_R	Ram drag	lb
$V_{T_{Std}}$	Standard true airspeed	ft/s
$V_{T_{Test}}$	Test true airspeed	ft/s

W	Weight	lb
W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb.

To apply this correction, the α_j variation from test day to standard day is needed. The climb angle, γ, and the thrust angle, α_j, both vary during the iteration process of determining standard day rate of climb. To simplify things, α_j is assumed to be small enough to neglect its effect.

7.5.1.4 ALTITUDE CORRECTION

The foregoing corrections allow P_s, in the form of rate of climb potential (i.e rate of climb corrected for increasing true airspeed) to be plotted for standard conditions. It may be necessary to refer these results to a new altitude. For example single-engine climb or wave-off performance at 5000 ft can be used to compute data for standard conditions at sea level. Constant weight and constant V_e correction are used to minimize the change in drag. An engine thrust model is used to calculate the change in net thrust due to changing altitude and temperature at constant V_e.

$$\Delta T_N = f\left(\Delta H_P, \Delta T_a\right) \tag{Eq 7.43}$$

Where:

T_N	Net thrust	lb
H_P	Pressure altitude	ft
T_a	Ambient temperature	˚C.

The rate of climb is computed for the new altitude using this thrust correction. This will again involve the climb angle iteration since the changes in rate of climb and true airspeed will change the climb angle, which will in turn affect the induced drag. These analytical corrections to thrust, and the iterative corrections to drag, can be minimized by performing the tests as close to the reference altitude as safety and operational restrictions permit. The corrected results are presented as shown in figure 7.23.

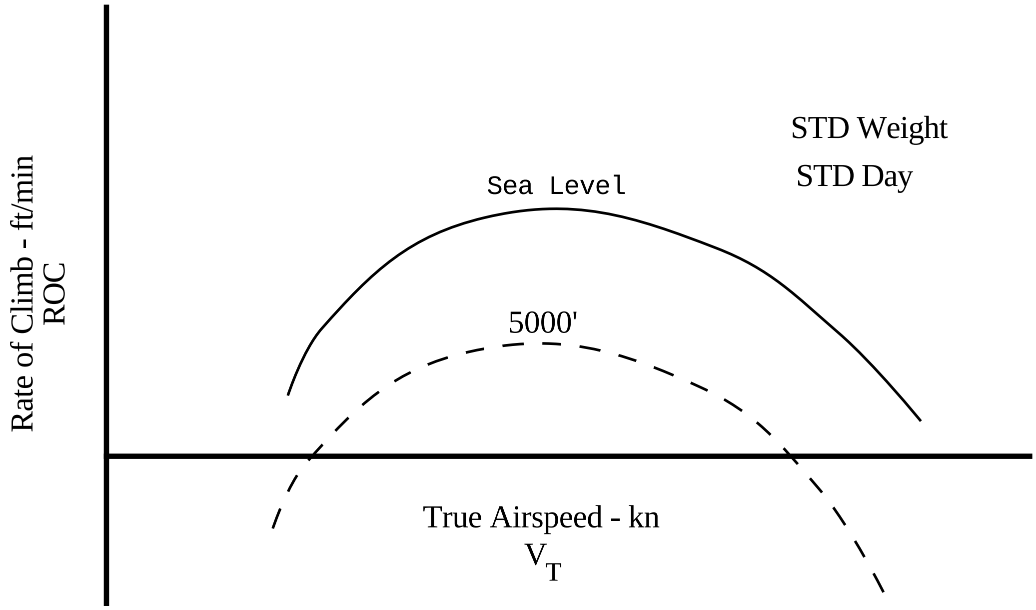

Figure 7.23
STANDARD DAY RATE OF CLIMB

7.5.2 COMPUTER DATA REDUCTION

Various computer programs are in existence to assist in reduction of performance data. This section contains a brief summary of the assumptions and logic which might be used. The treatment is purposefully generic as programs change over time or new ones are acquired or developed. Detailed instructions on the use of the particular computer or program are assumed to be available for the computer program. In any event, the operating system is invisible to the user. Data reduction from P_S level flight acceleration runs are in Chapter 5; however, energy analysis pertaining to climbs is reviewed.

7.5.2.1 ENERGY ANALYSIS

The purpose of the computer data reduction from energy analysis for climbs is to automatically calculate fuel, time and distance. This program is a subset to an energy analysis program which also calculates P_S from level acceleration runs. Basic data such as aircraft type, standard gross weight, etc. are entered.

Time to climb is calculated as follows:

$$\text{Time to Climb} = \int_{E_{h_1}}^{E_{h_2}} \frac{1}{P_s}\, dE_h \qquad \text{(Eq 7.25)}$$

The program must know $\frac{1}{P_s}$ as a function of E_h, therefore the program plots the function and asks for a curve fit. The curve appears similar to figure 7.24.

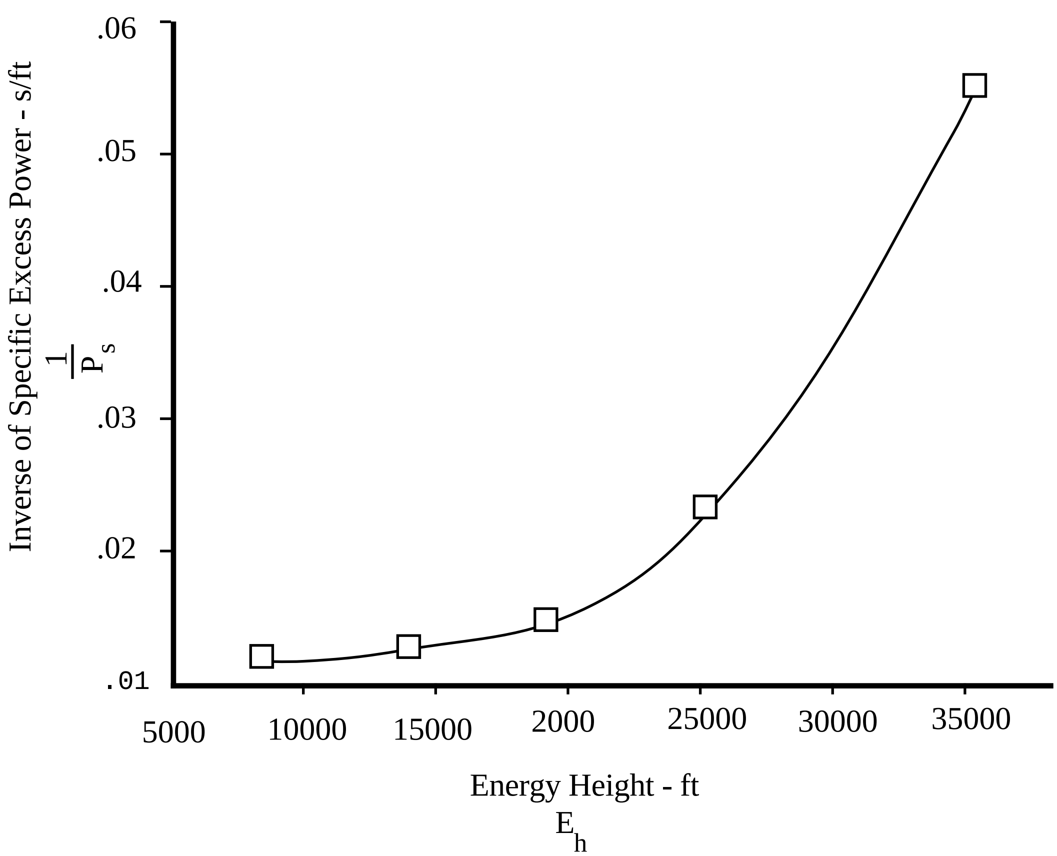

Figure 7.24
ENERGY RELATIONSHIP TO CLIMB

Distance flown in the climb is found by integration of the no wind ground speed with respect to time as in Eq 7.44.

$$\text{Distance} = \int_{t_1}^{t_2} V_T \cos\gamma \, dt \qquad \text{(Eq 7.44)}$$

To perform the integration, the program must know $V_T \cos\gamma$. The plot should appear similar to figure 7.25

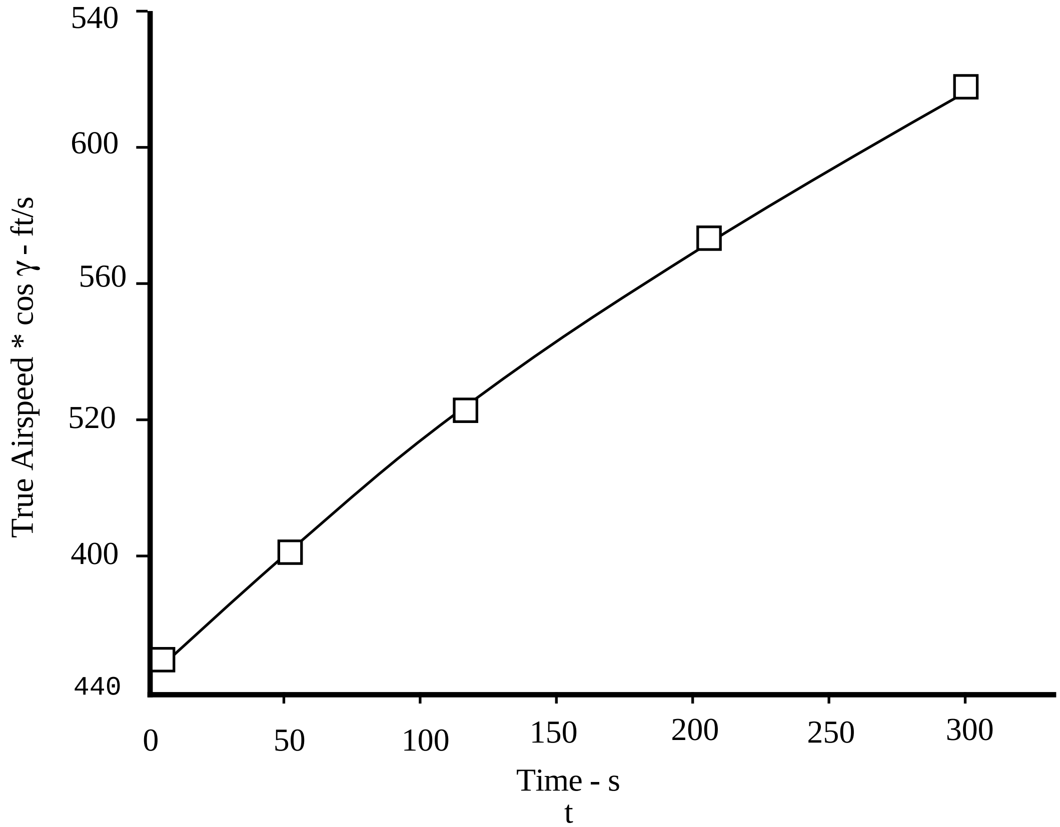

Figure 7.25
INTEGRATION RESULTS FOR DISTANCE

To calculate fuel used, the program must integrate standard day fuel flow with respect to time as in Eq 7.45.

$$\text{Fuel Used} = \int_{t_1}^{t_2} \dot{W}_f \, dt \qquad \text{(Eq 7.45)}$$

Where:

E_h	Energy height	ft
E_{h_1}	Energy height at start of climb	ft
E_{h_2}	Energy height at end of climb	ft
γ	Flight path angle	deg

P_S	Specific excess power	ft/s
t	Time	s
V_T	True airspeed	ft/s
$\dot{W}_f$	Fuel flow	lb/h.

The program first models the engine using referred fuel flow versus OAT, as in figure 7.26.

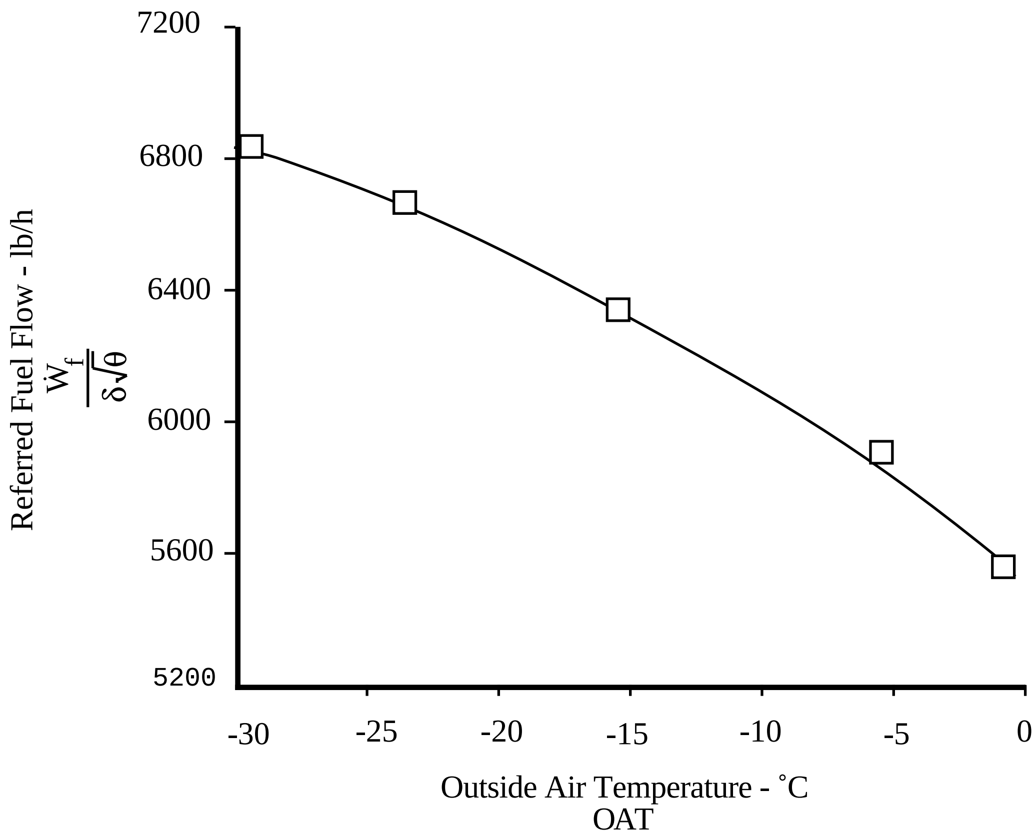

Figure 7.26
REFERRED FUEL USED

From figure 7.26, the program calculates standard day fuel flow and plots it versus time, as in figure 7.27.

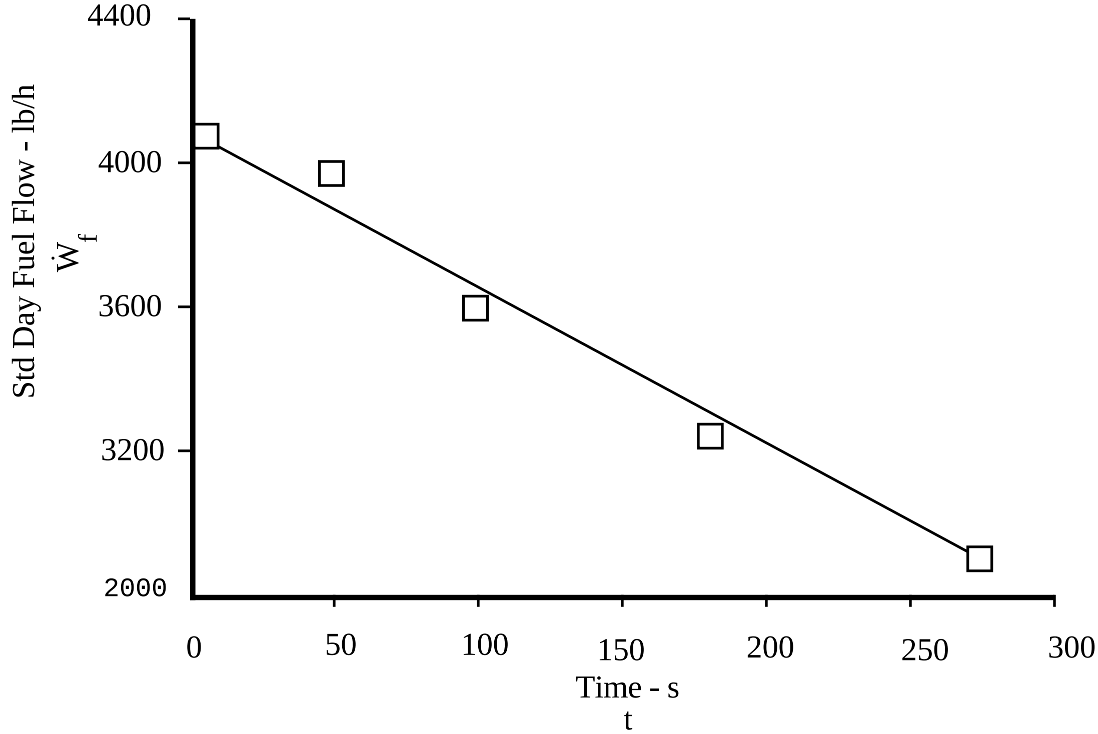

Figure 7.27
STANDARD DAY FUEL USED

The program then performs the three integrations using the curve fits determined, and plots altitude versus fuel used, time and distance in the climb. One possible example is shown in figure 7.28.

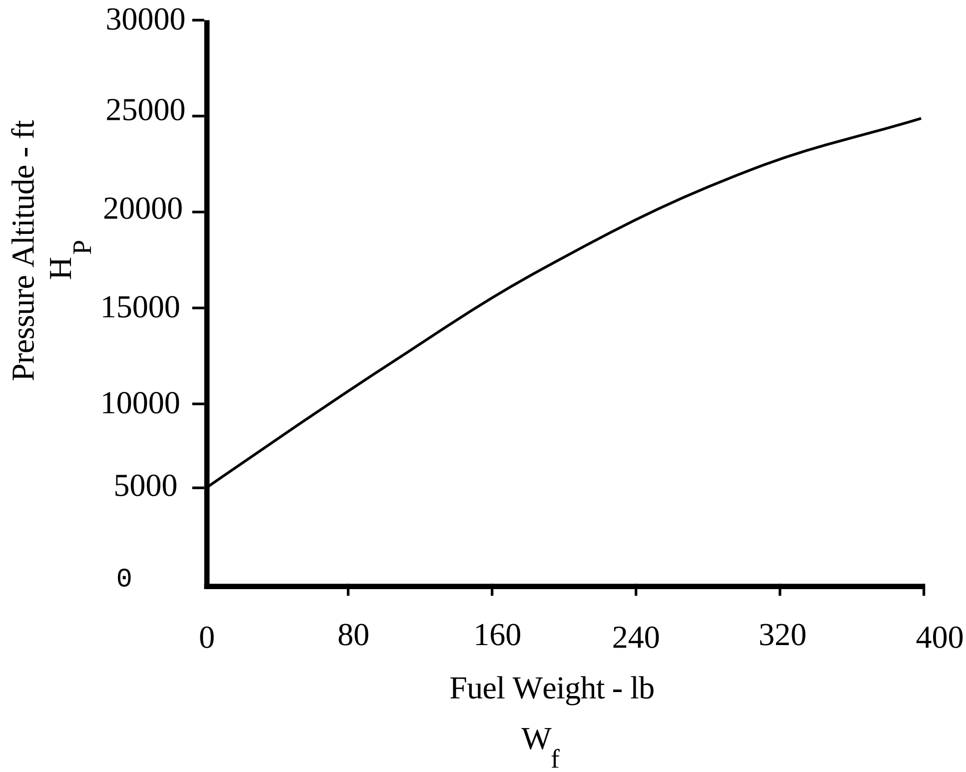

Figure 7.28
FUEL USED IN CLIMB

7.5.2.2 SAWTOOTH CLIMB

The purpose of the sawtooth climb data reduction program is to calculate rate and climb angle for any given gross weight, altitude and temperature, based on flight test data. From a menu selection, the appropriate choices are made to enter the sawtooth climb program. Data entry requirements for the program are as follows:

1. Basic data:
   a. Type of aircraft.
   b. Bureau number.
   c. Standard gross weight.
   d. Target altitude.
   e. Date of tests.
   f. Pilot’s name.
   g. Miscellaneous as allowed by the program.

2. For each data point:
   a. Initial indicated pressure altitude (ft).
   b. Final indicated pressure altitude (ft).
   c. Time required (s).
   d. Indicated airspeed (kn).
   e. OAT (˚C) or ambient temp (˚K).
   f. Fuel flow (lb/h).
   g. Gross weight (lb).
   h. Optional data as allowed by the program.

The program plots rate of climb and climb angle for the given altitude versus V_c, as in figures 7.29 and 7.30.

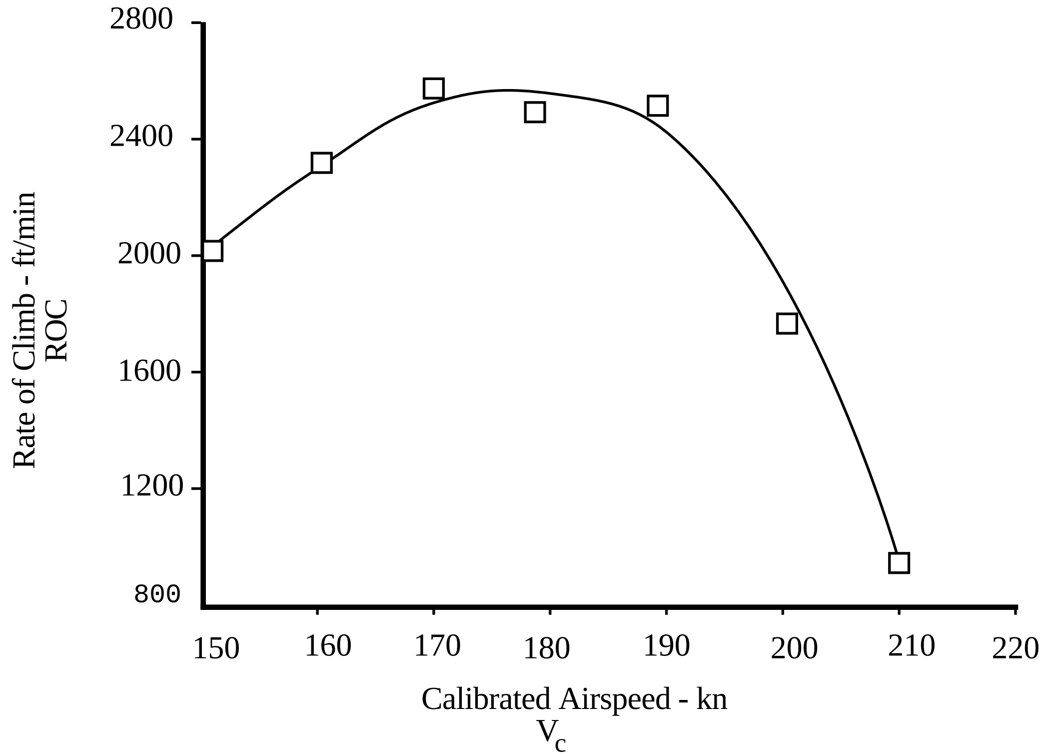

Figure 7.29

RATE OF CLIMB FROM SAWTOOTH CLIMBS

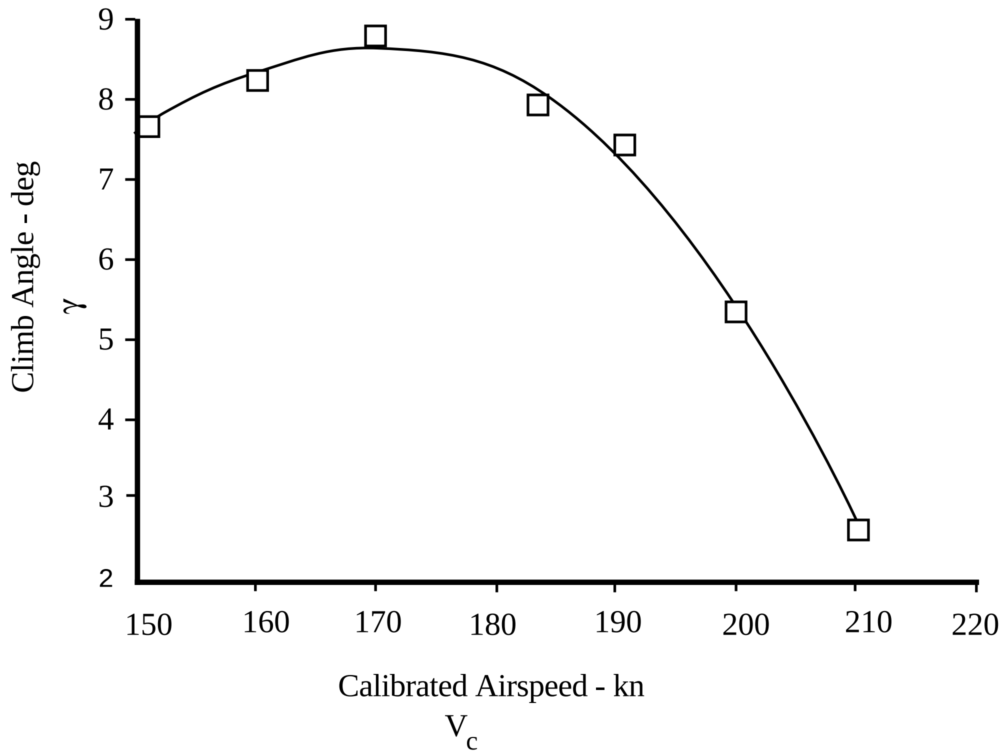

Figure 7.30
CLIMB ANGLE FROM SAWTOOTH CLIMBS

The following equations are used for the computer data reduction.

Obtain calibrated altitude (H_{P_c}) and calibrated airspeed (V_c) as in Chapter 2.

If ambient temperature (˚K) was entered:

$$°C = °K - 273.15 \quad \text{(Eq 7.46)}$$

$$OAT = f\left(T_a, M\right) \quad \text{(Eq 7.47)}$$

If OAT (˚C) was entered:

$$T_a = f\left(OAT, M\right) \quad \text{(Eq 7.48)}$$

True airspeed:

$$V_T = f\left(OAT, M_T\right) \quad \text{(Eq 7.49)}$$

Altitude:

$$h = H_{P_{c\,ref}} + \Delta H_{P_c}\left(\frac{T_a}{T_{std}}\right) \quad \text{(Eq 7.50)}$$

Energy height:

$$E_h = h + \frac{V_T^2}{2g} \quad \text{(Eq 7.1)}$$

Test day P_S:

$$P_s = \frac{dE}{dt}h = \frac{dh}{dt} + \frac{V_T}{g}\frac{dV_T}{dt} \quad \text{(Eq 7.2)}$$

Standard day P_S:

$$P_{s_{Std}} = P_{s_{Test}} \frac{W_{Test}}{W_{Std}} \frac{V_{T_{Std}}}{V_{T_{Test}}} + \frac{V_{T_{Std}}}{W_{Std}} \left(\Delta T_{N_x} - \Delta D\right) \quad \text{(Eq 7.32)}$$

Where:

D	Drag	lb
ΔD	Standard drag minus test drag	lb
ΔT_{N_x}	Standard net thrust parallel flight path minus test net thrust	lb
E_h	Energy height	ft
h	Tapeline altitude	ft
H_{P_c}	Calibrated pressure altitude	ft
$H_{P_{c\,ref}}$	Reference calibrated pressure altitude	ft
M	Mach number	

M_T	True Mach number	
OAT	Outside air temperature	˚C
$P_{s_{Std}}$	Standard specific excess energy	ft/s
$P_{s_{Test}}$	Test specific excess energy	ft/s
T_a	Ambient temperature	˚K
T_{N_x}	Net thrust parallel flight path	lb
T_{Std}	Standard temperature	˚K
V_T	True airspeed	ft/s
$V_{T_{Std}}$	Standard true airspeed	ft/s
$V_{T_{Test}}$	Test true airspeed	ft/s
W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb.

7.6 DATA ANALYSIS

The analysis of P_s data is directed towards determining the airplane optimum climb schedules. The theoretical optimum climb schedules are determined as described in section 7.3, by joining the points of maximum P_s (for a "minimum time" schedule) or points of maximum E_h (for a "maximum energy" schedule). The airspeed and Mach number schedules represented by these paths are then evaluated to determine whether they can be flown without undue difficulty, and are modified if necessary to make them flyable with the least penalty in climb performance. Finally, the climb schedules are flight-tested and the results, corrected to standard conditions, are compared with predictions.

The data from the sawtooth climb tests are intended to provide information for use in a far more restricted portion of the flight envelope than from level accelerations. The best climb speeds for the landing or single-engine wave-off configurations are unlikely to have much application above a few thousand feet, though they should certainly be determined for high elevation airports and should cover possible emergency diversion with the landing gear stuck down. The shape of these curves indicate the sensitivity of achieving the desired performance to airspeed errors. A peaked curve implies small inaccuracies in airspeed result in large performance penalties.

7.7 MISSION SUITABILITY

The mission requirements are the ultimate standard for climb performance. As stated in the beginning of the chapter, the mission of the airplane and specifically the mission of the flight to be flown determines the optimum condition for the climb. An interceptor is primarily interested in climbing to altitude with the minimum fuel if headed to a CAP station. An interceptor launching to intercept an incoming raid is primarily interested in arriving at the attack altitude with the best fighting speed and in the minimum time. An attack aircraft launching on a strike mission is primarily interested in climbing on a schedule of maximum range covered per pound of fuel burned. Still other types of mission may require, or desire, optimization of other factors during the climb.

In the case of single engine climb for multi engine aircraft or for wave-off, the test results in sawtooth climbs can develop the appropriate climb speeds to maximize obstacle clearance.

The specifications set desired performance in the production aircraft. These specifications are important as measures for contract performance. They are important in determining whether or not to continue the acquisition process at various stages of aircraft development.

As in all performance testing, the pure numbers cannot be the only determinant to acceptance of the aircraft. Mission suitability conclusions include the flying qualities associated with attaining specific airspeed schedules to climb. Consideration of the following items is worthwhile when recommending climb schedules or climb airspeeds:

1. Flight path stability.
2. Climb attitudes.
3. Field of view.
4. Mission profile or requirements.
5. Overall performance including climbing flight.
6. Compatibility of airspeeds / altitudes with the mission and location restrictions.
7. Performance sensitivities for schedule or airspeed deviations.

7.8 SPECIFICATION COMPLIANCE

Climb performance guarantees are stated in the detailed specification for the model and in Naval Air System Command Specification, AS-5263. The detail specification provides mission profiles to be expected and performance guarantees generically as follows:

1. Mission requirements.
   a. Land or sea based.
   b. Ferry capable.
   c. Instrument departure, transit, and recovery.
   d. Type of air combat maneuvering.
   e. Air to air combat (offensive and/or defensive) including weapons deployment.
   f. Low level navigation.
   g. Carrier suitability.

2. Performance guarantees are based on: type of day, empty gross weight, standard gross weight, drag index, fuel quantity and type at engine start, engine(s) type, loading, and configuration. The type of climb is specified, for example: Climb on course to optimum cruise altitude with military power (320 KIAS at sea level, 2 KIAS/1000 ft decrease to Mach number 0.72, Mach number 0.72 constant to level off).

Guarantees for climb likely include:

1. Instantaneous single engine climb in a given configuration.
2. A defined ceiling such as combat ceiling.
3. Wave-off rate of climb in a specific configuration.

AS-5263 further defines requirements for, and methods of, presenting characteristics and performance data for Naval piloted aircraft. Deviation from this specification are permissible, but in all cases must be approved by the procuring activity. Generally climb performance is presented as a function of altitude, plotting rate of climb at basic mission combat weight with maximum, intermediate, or normal thrust (power). Rates of climb for alternate loadings are presented to show the effects of drag changes with various external stores and/or weight changes. The effect of weight reduction during the climb is not considered. General provisions for the presentation of climb performance are as follows:

1. Performance is based on the latest approved standard atmospheric tables as specified by the Navy.

2. All speeds are presented as true airspeed in kn and Mach number; Mach number for jets, V_T for propeller aircraft.

3. Climb speed is the airspeed at which the optimum rate of climb is attained for the given configuration, weight, altitude, and power.

4. Service ceiling is that altitude at which the rate of climb is 100 ft/min at a stated loading, weight, and engine thrust (power).

5. Combat ceiling for subsonic vehicles is that altitude at which the rate of climb is 500 ft/min at the stated loading, weight, and engine thrust (power). Combat ceiling for supersonic vehicles is the highest altitude at which the vehicle can fly supersonically and have a 500 ft/min rate of climb at the stated loading, weight, and engine thrust (power).

6. Cruise ceiling for subsonic cruise vehicles is that altitude at which the rate of climb is 300 ft/min at normal (maximum continuous) engine ratting at stated weight and loading. Cruise ceiling for supersonic cruise vehicles is that altitude at which the rate of climb is 300 ft/min at normal (maximum continuous) engine ratting at stated weight and loading.

7. Cruise altitude is the altitude at which the cruise portion of the mission is computed.

8. Optimum cruise altitude is the altitude at which the aircraft attains the maximum nautical miles per pound of fuel for the momentary weight and configuration.

9. Combat altitude is the altitude at the target for the specific mission given.

10. Enroute climb data is based on the appropriate configuration, thrust (power) and weight. The aircraft has the landing gear and flaps retracted and the airspeed for best climb for the applicable condition is presented.

11. Enroute climb power for jet aircraft (fighter, attack, trainers) to cruise altitude is at intermediate (military) thrust. Propeller aircraft (patrol, transport) use maximum continuous power.

12. The time to climb to a specified altitude is expressed in minutes from start of enroute climb. Weight reduction as a result of fuel consumed is applied.

13. Combat climb is the instantaneous maximum vertical speed capability in ft/min at combat conditions; weight, configuration, altitude, and thrust (power).

14. The term thrust (power) is used to mean thrust (jet engine) and/or brake horsepower (shaft engines).

15. All fuel consumption data, regardless of source, is increased by 5 % for all engine thrust (power) conditions as a service tolerance to allow for practical operation, unless authorized otherwise. In addition, corrections or allowances to engine fuel flow is made for all power plant installation losses such as accessory drives, ducts, or fans.

7.9 GLOSSARY

7.9.1 NOTATIONS

AR	Aspect ratio	
a_x	Acceleration parallel flight path	ft/s^2
a_z	Acceleration perpendicular to flight path	ft/s^2
CAP	Combat air patrol	
CCF	Climb correction factor	
C_D	Drag coefficient	
C_L	Lift coefficient	
D	Drag	lb
d	Distance	ft
ΔD	Standard drag minus test drag	lb
dh/dt	Rate of climb	ft/s
D_{Std}	Standard drag	lb
D_{Test}	Test drag	lb
ΔT_{N_x}	Standard net thrust parallel flight path minus test net thrust	lb
e	Oswald's efficiency factor	
E_h	Energy height	ft
E_{h1}	Energy height at start of climb	ft
E_{h2}	Energy height at end of climb	ft
$E_{h_{Test}}$	Test energy height	ft
ESHP	Engine shaft horsepower	hp
g	Gravitational acceleration	ft/s^2
h	Tapeline altitude	ft
h_1	Tapeline altitude start of climb	ft
h_2	Tapeline altitude end of climb	ft

H_P	Pressure altitude	ft
H_{P_c}	Calibrated pressure altitude	ft
$H_{P_{c\ ref}}$	Reference calibrated pressure altitude	ft
H_{P_o}	Observed pressure altitude	ft
h_{Test}	Test tapeline altitude	ft
L	Lift	lb
M	Mach number	
M_T	True Mach number	
n_z	Normal acceleration	g
$n_{z_{Std}}$	Standard normal acceleration	g
$n_{z_{Test}}$	Test normal acceleration	g
OAT	Outside air temperature	˚C
P_A	Power available	ft-lb/s
P_a	Ambient pressure	psf
P_{req}	Power required	ft-lb/s
P_s	Specific excess power	ft/s
$P_{s_{Std}}$	Standard specific excess power	ft/s
$P_{s_{Test}}$	Test specific excess power	ft/s
ROC	Rate of climb	ft/s
S	Wing area	ft^2
t	Time	s
T_a	Ambient temperature	˚C
T_N	Net thrust	lb
T_{N_x}	Net thrust parallel flight path	lb
T_{Std}	Standard temperature	˚C
V	Velocity	ft/s
V_e	Equivalent airspeed	ft/s
V_H	Maximum level flight airspeed	kn
V_{hor}	Horizontal velocity	ft/s
V_i	Indicated airspeed	kn
V_{mc}	Airspeed for minimum control	kn
V_o	Observed airspeed	kn
V_s	Stall speed	kn
V_T	True airspeed	ft/s
$V_{T_{ref}}$	Reference true airspeed	ft/s
$V_{T_{Std}}$	Standard true airspeed	ft/s

$V_{T_{Test}}$	Test true airspeed	ft/s
V_v	Vertical velocity	ft/s
V_x	Speed for maximum climb angle	kn
V_y	Speed for maximum rate of climb	kn
W	Weight	lb
W_f	Fuel weight	lb
W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb
$\dot{W}_f$	Fuel flow	lb/h

7.9.2 GREEK SYMBOLS

α (alpha)	Angle of attack	deg
α_j	Thrust angle	deg
δ (delta)	Pressure ratio	
γ (gamma)	Flight path angle	deg
	Ratio of specific heats	
γ_{Std}	Standard flight path angle	deg
γ_{Test}	Test flight path angle	deg
π (pi)	Constant	
θ (theta)	Temperature ratio	
ρ_{ssl} (rho)	Standard sea level air density	0.0023769 slug/ft^3

7.10 REFERENCES

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